Complementarity and the Excess Burden of Taxation

I

Recent discussion on the respective merits of direct and indirect taxes has reached the point where Mr. Little has shown that, if the supply of labour is allowed to vary, the argument against indirect taxation is not perfectly general.[1] This article states the conditions where a change from an income tax to a system of indirect taxes, raising the same revenue from an individual, can increase the supply of effort and raise real income.

Our main, and simplest, model considers a consumer who is able to decide how much leisure he will take and how much he will work. All marginal costs are constant and competition is perfect. The consumer is initially in equilibrium buying three goods X, Y and L (leisure), and paying a flat rate income tax. We assume that all the consumer's income is spent and none saved. The rate of income tax is now slightly reduced and a small *ad valorem* indirect tax is introduced on one of the two goods, X or Y, so that the same tax revenue is raised from the consumer. We shall show that whenever this tax change makes the consumer work harder, he will reach a higher indifference surface ; whenever it means that he takes more leisure, his real income will fall. In general the consumer will work harder when the higher rate of tax is levied on that good (X or Y) which is " more complementary " with leisure, and *vice versa*. These conditions hold whatever the size and direction of income effects. The only exceptions can occur in a " crazy " case where tax rates are so high that an increase in the rate of tax on *one* good lowers the total yield of the tax system.

When we say that X is " more complementary " with leisure than Y we do not necessarily mean that X and leisure are complementary in the sense used by Professor Hicks.[2] Both could be competitive with leisure but, in some sense, X is " less competitive " (more complementary) than Y. Degrees of competitiveness are measured along a continuous scale from very competitive goods at one end to very complementary goods at the other. The exact way in which this is done will be explained later.

The main analysis considers small changes in tax rates and does not indicate the size of the movements away from the initial equilibrium position needed to obtain an " optimum " system of taxation. This problem will be discussed briefly, together with the effects of the relaxation of some of our assumptions, towards the end of the article.

II

Let us consider an individual consumer who is in equilibrium, paying a flat-rate income tax at the rate $\frac{r}{1+r}$. He has acquired those amounts of three goods, X, Y and L which give maximum satisfactions from his post-tax income. Leisure is considered as a good providing satisfactions just like X and Y. Because of the difficulty of measur-

[1] Haskell P. Wald, " The Classical Indictment of Indirect Taxation," *Quarterly Journal of Economics*, 1944–45, p. 577.
A. Henderson, " The Case for Indirect Taxation," *Economic Journal*, 1948, p. 538.
I. M. D. Little, " Direct versus Indirect Taxes," *Economic Journal*, 1951, p. 577.

[2] J. R. Hicks, " A Reconsideration of the Theory of Value," *Economica*, 1934, pp. 69–71.

ing leisure, our algebraic analysis is formulated in terms of X, Y and W (work). In other words, we consider hours of work as numbers of hours not taken as leisure and *vice versa*.

The formula for the rate of income tax may look cumbersome. It is given this particular value because we follow the usual procedure of regarding an income tax as the equivalent of a purchase tax levied at the same *ad valorem* rate on all goods, except leisure. Purchase taxes are usually calculated as a proportion of price *excluding* tax. A direct tax on a commodity at the rate $r = \frac{1}{3}$, would thus be called, in the real world, a $33\frac{1}{3}$ per cent purchase tax. Income taxes, however, are usually expressed as a percentage of pre-tax income. Therefore, an *ad valorem* tax of $33\frac{1}{3}$ per cent levied on all goods (except leisure) is the equivalent, for the tax payer, of an income tax. But the rate of this equivalent income tax is called 25 per cent and not $33\frac{1}{3}$ per cent. The initial income tax can therefore be considered as an *ad valorem* tax on all goods and since the rate of *ad valorem* tax is r, the corresponding income tax rate is $\frac{r}{1 + r}$.

This *ad valorem* tax, levied at the equal rate of r on both X and Y, is now replaced by *ad valorem* taxes on X and Y levied at the *unequal* rates of r_1 and r_2 respectively. These rates are fixed so that the total tax paid remains the same (in money terms), after the change takes place. It follows that the *ad valorem* rate of tax on one of the two goods, X or Y, will normally be lower than in the original situation where the income tax was simulated by the tax at the rate r on both X and Y. Similarly, the *ad valorem* rate of tax on the other good will be greater than initially. Symbolically, either $r_1 > r > r_2$; or $r_1 < r < r_2$.[1]

When the tax change occurs there will be an income effect and a substitution effect. The income effect will cause the consumer's real income to rise or fall—he will reach a higher or a lower indifference surface. The income and substitution effects combined will determine whether the consumer will hold more or less of the goods, X, Y and L with the indirect tax than with the income tax.

The term " income effect " is not used here in quite its normal sense. An income effect shows the influence on a consumer's satisfactions and on his purchases of goods of an increase in his money income—all prices being constant. It does not normally take a change in the amount of his leisure into account. But the income effects in this article show the change in the consumer's holdings of X, Y and L, when he is given an increment of income which does not depend on the amount of work he does. They show the effects of a " poll subsidy," increased family allowances or increased investment income, for example. It is clearly realistic to look on income effects in this way. A consumer's satisfactions can be altered just as much by a change in the amount of his leisure as by a change in his purchases of consumer goods. We shall use the term " income effect " in this special sense to include any effect on the amount of leisure which a consumer takes.

We can now tackle our first main problem. Under what conditions will the introduction of unequal tax rates (the tax paid remaining constant) alter the amount of work done by a consumer ?

Suppose that when the *ad valorem* rates on X and Y are equal the prices of X and Y (before tax) are p_1 and p_2, and after tax are P_1 and P_2. The tax revenue is then $X(P_1 - p_1) + Y(P_2 - p_2)$. When the tax change takes place, these prices become $P_1 + dP_1$ and $P_2 + dP_2$. Since we assume constant marginal costs and perfect com-

[1] These conditions will not always hold. In the " crazy " case mentioned in Section I, r might not lie between r_1 and r_2.

petition, p_1 and p_2 do not alter. So, the change in revenue, caused by the tax change is :

$$\left[(P_1 - p_1)\frac{\partial X}{\partial P_1} + (P_2 - p_2)\frac{\partial Y}{\partial P_1} + X\right] dP_1$$
$$+ \left[(P_1 - p_1)\frac{\partial X}{\partial P_2} + (P_2 - p_2)\frac{\partial Y}{\partial P_2} + Y\right] dP_2 \quad \ldots\ldots (1)$$

Since in the initial " quasi income tax " position the rate of tax on X and Y is the same, this reduces to :

$$\left[p_1 r\frac{\partial X}{\partial P_1} + p_2 r\frac{\partial Y}{\partial P_1} + X\right] dP_1$$
$$+ \left[p_1 r\frac{\partial X}{\partial P_2} + p_2 r\frac{\partial Y}{\partial P_2} + Y\right] dP_2 \quad \ldots\ldots (2)$$

and, as we assume that tax revenue remains constant when the tax system changes, this must equal zero.

Let U (X, Y, W) be a utility function. In any equilibrium, where a tax is being levied, the consumer maximises this function, subject to the constraint :

$$M + W = XP_1 + YP_2$$

where W is work done, measured in pounds earned, and M is " unearned " income. If M is positive it represents the " poll subsidy " mentioned above ; if M is negative, it represents a poll tax. In the usual way, the conditions for maximising satisfaction are :

$$\begin{aligned} XP_1 + YP_2 - W - M &= 0 \\ U_x - \lambda P_1 &= 0 \\ Y_y - \lambda P_2 &= 0 \\ U_w + \lambda &= 0 \quad \ldots\ldots (3) \end{aligned}$$

where λ, the Lagrange multiplier, is the marginal utility of money.

The price changes in which we are interested are a change of P_1 to $P_1 + dP_1$ and of P_2 to $P_2 + dP_2$. The resulting change in the amount of work done, dW, is given by :

$$dW = \frac{\partial W}{\partial P_1} dP_1 + \frac{\partial W}{\partial P_2} dP_2 \quad \ldots\ldots (4)$$

From the first relation in (3), it follows that :

$$P_1 \frac{\partial X}{\partial P_1} + P_2 \frac{\partial Y}{\partial P_1} - \frac{\partial W}{\partial P_1} + X = 0$$
$$\text{and} \quad P_1 \frac{\partial X}{\partial P_2} + P_2 \frac{\partial Y}{\partial P_2} - \frac{\partial W}{\partial P_2} + Y = 0 \quad \ldots\ldots (5)$$

Since $P_1 = p_1 (1 + r)$ and $P_2 = p_2 (1 + r)$ we can use (5) to eliminate $\frac{\partial X}{\partial P_1}, \frac{\partial X}{\partial P_2}, \frac{\partial Y}{\partial P_1}$, and $\frac{\partial Y}{\partial P_2}$ from (2). We then have as the change in tax revenue :

$$\left[\frac{r}{1 + r}\frac{\partial W}{\partial P_1} + \frac{1}{1 + r}X\right] dP_1 + \left[\frac{r}{1 + r}\frac{\partial W}{\partial P_2} + \frac{1}{1 + r}Y\right] dP_2 \quad \ldots\ldots (6)$$

In order that the total tax revenue should be unchanged we thus require the changes in the two prices to satisfy :

$$\left[r\frac{\partial W}{\partial P_1} + X\right] dP_1 + \left[r\frac{\partial W}{\partial P_2} + Y\right] dP_2 = 0 \quad \ldots\ldots (7)$$

Substituting in (4) we have :

$$dW = \left[\frac{\partial W}{\partial P_1} - \frac{r\dfrac{\partial W}{\partial P_1} + X}{r\dfrac{\partial W}{\partial P_2} + Y} \frac{\partial W}{\partial P_2} \right] dP_1 = \left[\frac{Y\dfrac{\partial W}{\partial P_1} - X\dfrac{\partial W}{dP_2}}{r\dfrac{\partial W}{\partial P_2} + Y} \right] dP_1$$

Therefore :

$$dW = \left[\frac{YV_{wx} - XV_{wy}}{r\dfrac{\partial W}{\partial P_2} + Y} \right] dP_1 \qquad \ldots\ldots (8)$$

Similarly, expressing dW in terms of the change in the price of Y :

$$dW = \left[\frac{XV_{wy} - YV_{wx}}{r\dfrac{\partial W}{\partial P_1} + X} \right] dP_2$$

In these equations V_{wx} and V_{wy} are the substitution terms in the Slutsky equation which shows the change in the amount of work done by the consumer resulting from changes in the prices of X and Y respectively. This change in the amount of work done (dW) will be positive whenever the expression on the right hand side of equation (8) is positive.

The denominator in this expression can only be negative in our " crazy " case. For the denominator is equal to $(1 + r)$ times the rate of change of tax receipts resulting from an increase in the tax on Y (with no change in the tax on X). It will only be negative if this change in the tax receipts is negative, and our crazy case occurs where an increase in the tax on one good, with no change in the tax on the other, *will* reduce tax receipts. We shall, therefore, ignore for the present the possibility that the denominator might be negative.

The expression $YV_{wx} - XV_{wy}$ in equation (8) will be positive if, in some sense, Y is more complementary with work than X is (or if X is more complementary with leisure). dW will then have the same sign as dP_1. If there are three goods, X, Y and L, a consumer will always work harder as the result of the introduction of the indirect tax (total tax paid remaining constant) if it is levied on *that good* (X *or* Y) *which is more complementary with leisure.* For example, suppose that the indirect tax is levied on X so that $r_1 > r_2$. If X is more complementary with L than Y is, the consumer will take less leisure in the indirect tax equilibrium. He will work harder.

Let us now say precisely what we mean when we say that X is more complementary with leisure than Y is. A greater or smaller degree of complementarity could easily be defined if it were possible to measure the quantity of leisure, as could be done if there were a maximum to the income which could be earned however hard an individual were to work. The quantity of leisure could then be measured by the difference between this maximum income and his actual earnings. In that case, our condition would be that the elasticity of complementarity[1] between X and leisure should be higher than the elasticity of complementarity between Y and leisure.

We can now proceed to show in what conditions the tax change will raise the consumer's real income by putting him on a higher indifference surface.

[1] Cf. R. G. D. Allen, " A Reconsideration of the Theory of Value," *Economica*, 1934, pp. 205–6.

The change in the utility function (dU) is given by :

$$dU = \left[U_x\frac{\partial X}{\partial P_1} + U_y\frac{\partial Y}{\partial P_1} + U_w\frac{\partial W}{\partial P_1}\right] dP_1 + \left[U_x\frac{\partial X}{\partial P_2} + U_y\frac{\partial Y}{\partial P_2} + U_w\frac{\partial W}{\partial P_2}\right] dP_2$$

$$= \lambda\left[P_1\frac{\partial X}{\partial P_1} + P_2\frac{\partial Y}{\partial P_1} - \frac{\partial W}{\partial P_1}\right]dP_1 + \lambda\left[P_1\frac{\partial X}{\partial P_2} + P_2\frac{\partial Y}{\partial P_2} - \frac{\partial W}{\partial P_2}\right] dP_2$$

(from equation (3))

Therefore $dU = -\lambda\,(XdP_1 + YdP_2)$ (from equation (5))

$$= +\lambda\left(r\frac{\partial W}{\partial P_1}dP_1 + r\frac{\partial W}{\partial P_2}dP_2\right) \text{ (from equation (7))}$$

$$= r\lambda dW \text{ (from equation (4))}$$

where dU is the increment of the utility index (the increase in satisfaction) ; r is the *ad valorem* rate of purchase tax in the initial " quasi income tax " equilibrium ; λ is the Langrange multiplier (the " marginal utility of money ") ; and dW is the increment of work done by the consumer, measured in money earned.

The consumer will only reach a higher real income as a result of the tax change—the increment in the utility function will only be positive—if $r\lambda dW$ is positive. Now r is the rate of tax and must be positive ; so must λ, the marginal utility of money. The consumer will, therefore, have a higher real income whenever dW is positive—whenever he works harder as a result of the tax change.

The above equation also shows that the rise in real income is equal to the marginal utility of the increase in money income, multipied by the *ad valorem* tax rate in the initial equilibrium. Thus for any given increase of work, the resulting rise in the consumer's real income will be larger the greater the marginal utility of money and the higher the *ad valorem* rate of tax.

Although this formula is simple, its meaning is not immediately obvious. One way of looking at it is this. If there had been no change in leisure, the consumer's money income would have been constant and the change in the tax system would have reduced his real income. This is shown by the traditional indifference curve diagram,[1] where a change from direct to indirect taxes (with money income and the tax yield constant) always leaves the consumer worse off. But if the consumer works harder after the tax change, his money income rises. In our model his increased income is dW and its utility λdW. Not all the satisfactions from spending this extra income represent a net gain to the consumer. Some of the utility derived from the extra income is cancelled out by the reduction in the amount of leisure now that the consumer works harder.

Again, some utility derived from the extra income has to make good the loss of satisfaction which would have occurred if the tax change had led to no increase in work. This latter loss of satisfaction is the one usually studied in this kind of analysis, but it is only a second order change (i.e. $dU = 0$, $d^2U < 0$) for an infinitesimal change like that we are studying, and we can ignore it.

The reason why the gain in satisfaction is only $r\lambda dW$ is, therefore, as follows. If the consumer is in the income tax equilibrium, a move away from that equilibrium position along the budget plane (including work) would cause a loss of satisfaction, but only of the second order. In order to buy the extra amounts of X and Y taken in the indirect tax equilibrium, the consumer would have to supply extra work equal to $(1 + r)\,dW$. With the indirect tax, however, the necessary extra work is only dW.

[1] Cf. for example, Henderson, op. cit., p. 540, and Little, op. cit., p. 577.

(Since no extra tax revenue is needed, expenditure and income increase by equal amounts.) There is thus a net gain of $r\lambda dW$, over a movement along the old budget plane. Our results for infinitesimal changes show the direction in which the tax system should change if real income is to rise. With finite changes, the loss of satisfaction caused by movements along the old budget plane would no longer be negligible. To find the " optimum " indirect tax system, we should have to take account of these second order changes along the old budget plane.

We cannot say with certainty what the effect of the tax change on the purchases of X and Y will be. This will depend both on the actual magnitudes of the substitution terms and on the income effects.

An illustration may make the position clearer. Let us suppose that the three goods, X, Y and L are cricket matches, food and leisure respectively. We assume that cricket matches[1] are more complementary with leisure than food is. We further assume that when the change takes place, the *ad valorem* rate of tax on cricket matches is higher than in the " quasi income tax " equilibrium and the rate of tax on food is lower. The tax change will thus make the consumer work harder. He will do more work and his real and money incomes will rise. Normally he will buy more food and watch less cricket, but neither result is certain.

The *rationale* of such a change from direct to indirect taxation is quite simple. The " ideal " tax would be one which was levied at the same *ad valorem* rate on all goods, including leisure.[2] By taxing those goods which are more complementary with leisure, one is to some extent taxing leisure itself. One is, therefore, moving in the direction of the " ideal " tax which is levied at equal *ad valorem* rates on all goods. This explains why the consumer reaches a higher indifference surface. We may also note that, since the total tax payment is constant, a smaller proportion of the consumer's money income is paid in taxes when he works harder. This provides part of the reason why more work raises the consumer's real income. The relative simplicity of the conditions under which real income rises or falls is the result of the constraint introduced by the central assumption of constant tax payments.

It is also possible to use our results to show whether one tax system can raise more revenue than another, from a given consumer on a given indifference surface. It can be shown that an indirect tax can raise more revenue, at a given level of real income, than an income tax, if the *ad valorem* rate of tax is higher on that good which is more complementary with leisure.

In discussing this subject, one finds that confusion sometimes arises because of attempts to divide leisure into two parts, that used in conjunction with other goods, for example, in listening to symphony concerts, and that used as " pure leisure "—when one does nothing at all. This distinction is a dangerous one. But even if the distinction is valid, the conclusions of the foregoing analysis are unaffected. An increase in, say, theatre prices, may induce a consumer to spend time which he previously spent at the theatre in " doing nothing." But since prices of goods like food are now lower, extra work must become a less unpleasant alternative to leisure. In this simple model, the consumer may spend more time " just sitting " in the garden as a result of the tax change. But, at least, he will spend some time he previously used in watching plays in doing a little extra work.

We can now show what change in the relative prices of X and Y is needed to

[1] Cf. Henderson, op. cit., p. 546.
[2] This is essentially equivalent to a poll tax.

keep the tax paid constant. We have seen from (7) that the ratio between the two price changes :

$$\frac{dP_2}{dP_1} = -\frac{r\frac{\partial W}{\partial P_1} + X}{r\frac{\partial W}{\partial P_2} + Y} \quad \text{(9)}$$

therefore : $$\frac{dP_2}{dP_1} = -\frac{-rX\frac{\partial W}{\partial M} + X + rV_{wx}}{-rY\frac{\partial W}{\partial M} + Y + rV_{wy}}$$

$$= -\frac{X}{Y} - \frac{r}{Y}\frac{YV_{wx} - XV_{wy}}{Y\left(1 - r\frac{\partial W}{\partial M}\right) + rV_{wy}} \quad \text{(10)}$$

The change in the price of Y is greater relatively to the change in the price of X the greater is :

$$\frac{r}{Y} \cdot \frac{YV_{wx} - XV_{yw}}{Y\left(1 - r\frac{\partial W}{\partial M}\right) + rV_{wy}}$$

That is to say, the price of Y will change more, relatively to the price of X, if X is more complementary with leisure, than Y is. The greater the difference between the complementarity of the two goods with leisure, the greater this relative price change.

Let us now consider the " crazy " case mentioned earlier. Here either the numerator or the denominator, or both, in the right hand side of equation (9) are negative. The numerator will be negative if an increase in the rate of tax on X (with the tax on Y constant) lowers the total tax yield. Similarly, the denominator will be negative if an increase in the rate of tax on Y has this result. These results can only occur if (*i*) the tax rates are high, and *either* (*ii*) leisure is an inferior good, *or* (*iii*) the good in question is so strongly competitive with leisure (in Hicks' sense) that the substitution effect outweighs the (positive) income effect. It would obviously be ridiculous to allow such a situation to persist, since a tax reduction would raise the revenue received by the Exchequer. Nevertheless, such a situation might conceivably occur and is worth analysing.

Let us first see what happens where the " crazy " case is reached for Y but not for X. This can only happen, whether leisure is an inferior good or not, if X is more complementary with leisure than Y is. To obtain the same tax revenue when a change from direct to indirect taxation occurs, both prices have to change in the same direction. An *increase* in tax rates would inevitably mean less work done and lower real incomes. A *reduction* in tax rates would increase both the supply of effort and real income. Similarly when the " crazy " case has been reached for X but not for Y.

When the " crazy " situation exists for both goods, the normal rules are reversed whether or not leisure is an inferior good. If tax revenue is to be constant, the tax rates will have to change in opposite directions. But the consumer will only work harder if the higher *ad valorem* tax rate is levied on the good which is *less complementary* with leisure. The consumer will then become " better off " in the usual way.

An illustration of a position where both X and Y are " crazy " may be useful. The notion that leisure might be an inferior good is improbable so we ignore it. If the " crazy " situation has nevertheless been reached for both X and Y, it follows that an increase in the tax rate on either good alone will reduce both the total tax yield and the supply of effort. If there is a rise in the rate of tax on the good which is more complementary with leisure, however, a greater part of the loss of tax revenue resulting from the reduction in work will be offset by the increased yield on the good in question.

The problem we are considering occurs where the two tax rates change in opposite directions and give the same yield. Here, a small increase in the tax on the good which is less complementary with leisure means the loss of much tax revenue and to offset this there must be a relatively large reduction in the tax on the other good. It is the relative largeness of this second change which causes the net effect to be favourable.

These exceptions to our main conclusions should be noted. The main argument relates to a model where tax rates are moderate. It should also be remembered that a change to indirect taxation *cannot* raise the consumer's real income if his supply of effort has zero elasticity, or if there is no additional work available. This should be borne in mind in any attempt to relate our conclusions to present day conditions.

Some of the restrictions in the analysis can now be relaxed. First, what happens where there are more than two consumer goods as well as leisure? We can again consider the initial income tax position by assuming that an *ad valorem* tax at the rate r is levied on all goods except leisure. The tax change can now be looked upon as a change in the rate of tax on any two of these goods, say, X_1 and X_2, the money tax yield being unaltered.

Equation (2), showing the necessary relative price-changes, is modified simply by including similar terms for the other goods in each of the brackets. The same modification has to be made in equation (5), but the result is that the relationship shown in equation (7) remains:

$$\left[r\frac{\partial W}{\partial P_1} + X\right] dP_1 + \left[r\frac{\partial W}{\partial P_2} + Y\right] dP_2 = 0.$$

The remainder of the argument applies as before. Where the tax rate is changed on more than two out of any collection of goods, our conclusions still hold, provided that the "average" (in some sense) rate of tax is raised on those goods which are most complementary with leisure.

The same type of analysis can be applied for a tax change from any initial tax system, starting from a modified form of equation (2) with differing values for r for the various terms. Putting $dU = 0$, or $XdP_1 + YdP_2 = 0$, and similar relations for changes affecting other pairs of goods, we can obtain the formal conditions for the "optimum" system of indirect taxes. The solution gives a system of equations in the n tax rates with coefficients involving the elasticities of complementarity between all pairs of goods. It does not follow, however, that goods most complementary with leisure would then bear the highest rate of tax. For example, if one of the goods had both zero income and substitution elasticities, the "optimum" position would be where the whole tax was raised from that good.

By assuming constant marginal costs our analysis has ignored the effects of changing marginal rates of transformation. We have assumed away the difficulties which arise because the Treasury does not want to acquire money from tax payers, but real resources. Since, in practice, marginal rates of transformation are bound to change, this problem is important if we are to make the transition from one to many consumers. We make this transition now.

Let us follow Mr. Little[1] in constructing a transformation surface showing those combinations of the various goods and leisure (or work) which remain to consumers

[1] Op. cit., pp. 581–3.

after the Government has taken all the goods it requires. We can express the nature of this transformation surface by the function :

$$\phi\ (X_1,\ X_2\ .\ .\ .\ X_n,\ W) = 0.$$

Let us now consider a change from an income tax situation. The condition that in any equilibrium position the Government has a given collection of goods, takes the place of our assumption that the total tax paid is constant.

It follows that the changes in the quantities of goods, and in work done must satisfy the relation :

$$\frac{\partial\phi}{\partial X_1}\,dX_1 + \frac{\partial\phi}{\partial X_2}\,dX_2\ .\ .\ .\ \frac{\partial\phi}{\partial X_n}\,dX_n + \frac{\partial\phi}{\partial W}\,dW = 0.$$

Consider again the case where the tax rate changes on the goods X_1 and X_2 only. We then have :

$$dP_1\left[\frac{\partial\phi}{\partial X_1}\frac{\partial X_1}{\partial P_1} + \frac{\partial\phi}{\partial X_2}\frac{\partial X_2}{\partial P_1} + \dots \frac{\partial\phi}{\partial X_n}\frac{\partial X_n}{\partial P_1} + \frac{\partial\phi}{\partial W}\frac{\partial W}{\partial P_1}\right] +$$
$$dP_2\left[\frac{\partial\phi}{\partial X_1}\frac{\partial X_1}{\partial P_2} + \frac{\partial\phi}{\partial X_2}\frac{\partial X_2}{\partial P_2} + \dots \frac{\partial\phi}{\partial X_n}\frac{\partial X_n}{\partial P_2} + \frac{\partial\phi}{\partial W}\frac{\partial W}{\partial P_2}\right] = 0 \quad \dots\dots\dots\dots (11)$$

In the initial equilibrium position with the income tax, the ratios of the partial derivatives of ϕ give the marginal rates of transformation between the different goods. Since we are assuming perfect competition, these marginal rates of transformation will be equal to relative prices before tax. That is to say :

$$\frac{\partial\phi}{\partial X_1} : \frac{\partial\phi}{\partial X_2} : \dots \frac{\partial\phi}{\partial X_n} : \frac{\partial\phi}{\partial W} = p_1 : p_2 : \dots p_n : -1 = P_1 : P_2 : \dots P_n : -(1+r)$$

Substituting this into equation (11) we have :

$$dP_1\left[P_1\frac{\partial X_1}{\partial P_1} + P_2\frac{\partial X_2}{\partial P_1} + \dots P_n\frac{\partial X_n}{\partial P_1} - (1+r)\frac{\partial W}{\partial P_1}\right] +$$
$$dP_2\left[P_1\frac{\partial X_1}{\partial P_2} + P_2\frac{\partial X_2}{\partial P_2} + \dots P_n\frac{\partial X_n}{\partial P_2} - (1+r)\frac{\partial W}{\partial P_2}\right] = 0.$$

It follows from an equation similar to that given in (5) that :

$$dP_1\left[-r\frac{\partial W}{\partial P_1} - X\right] + dP_2\left[-r\frac{\partial W}{\partial P_2} - Y\right] = 0.$$

This, once again, is the condition we had in (7).

It follows that changing marginal costs do not invalidate the conditions which we have outlined. The community will work more, and will have a higher real income, under exactly the same conditions as for the individual consumer. But the *final equilibrium position* is likely to differ.

What has been said above relates, strictly, only to the respective effects of indirect taxes and of proportional, " flat rate," income taxes. In the real world, however, a " progressive " income tax system usually includes a fixed allowance, granted by exempting a certain initial income from tax. Once the exemption limit is passed the marginal rate of tax usually changes only discontinuously so that over considerable ranges of income the " progressive " income tax really represents a flat rate tax plus a " poll subsidy." For small changes from this position towards indirect taxation, the foregoing analysis appears to be valid.[1]

[1] It might be worth mentioning that the case cited by Mr. Wald (op. cit., pp. 588–9), where a shift from a proportional to a progressive income tax makes the consumer better off, is the equivalent of our " crazy " case. As shown in his diagram, a lower marginal rate of tax, together with tax free allowances is raising as much revenue as a higher flat rate tax.

An interesting point emerges if one considers a true progressive tax. Mr. Henderson suggested that one way to compare direct and indirect taxes was to discover that system of indirect taxes which yielded, at each level of income, the same amount of revenue as a progressive income tax.[4] He then showed that, *if money income is held constant* (if leisure is held constant) the direct tax proves " superior " in the conventional sense of this term. But if leisure is *not* held constant, we know that the indirect tax can be the " superior " tax. The comparison of direct and indirect taxes with the same " tax formula " then becomes difficult. For, one reason why a change from direct to indirect taxation can increase a man's real income, is that it reduces the proportion of his income paid in tax. In other words, it reduces the progressiveness of the tax system. Mr. Henderson is, therefore, right, in a sense, when he claims that " it is not the change of method which is causing the change in output but the change in the tax formula from the point of view of the tax payer."[2] Once one moves away from a two-dimensional world, however, it is no longer true that there will be a direct tax system which has the same formula from the point of view of the Exchequer and is always " less onerous to the payer."[3]

Mr. Henderson is perfectly correct in claiming that a direct tax system can always raise a *given* amount of revenue from a *given* money income more painlessly than an indirect tax. But once the amount of leisure is allowed to vary, money income will vary too. The best way in which the problem can then be approached is by discovering which tax system raises a given revenue most efficaceously from a given " potential income " or a given " income earning capacity." It will then be found that a direct tax system will nearly always impose an " excess burden " as compared with some system of indirect taxes.

Let us now sum up the argument and consider some implications of our results. We have seen that some form of indirect taxation will be superior to direct taxation *if individuals are able to decide how much they work.* The discovery of the goods on which indirect taxes are required remains unsolved until we have more adequate statistical information about demand equations. Only in particular cases—such as the practically unimportant one of *completely* inelastic demand—can one make dogmatic statements about the " optimum " system of indirect taxation.

It should, however, be made quite clear that nothing in our analysis conflicts with the acknowledged superiority of the poll tax over all other forms of tax. Nor, of course, does anything in this article deny that a change from direct to indirect taxation might well reduce the progressiveness of the tax system, without enabling anyone to prove that it was less progressive. Such a proof is obviously impossible because no one knows the shape of all indifference maps.

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[1] Op. cit., pp. 541–3.
[2] Op. cit., p. 545.
[3] Op. cit., p. 544.