数字图像处理
(**D**igital **I**mage **P**rocessing)

图像插值

图像插值

- **图像插值的应用**

  - **Want "BIG" images**
   - 当我们需要放大一幅图像去观察某一细节时，需要确定给定的新位置的像素灰度

  - **Want "GOOD" images**
    - 数字图像某部分缺失或损坏，需要去估计缺失的像素点灰度

  - **Want "COOL" images**
    - 需要特殊艺术效果时，需要去估计仿射变换后像素点的灰度

BIG IMAGES
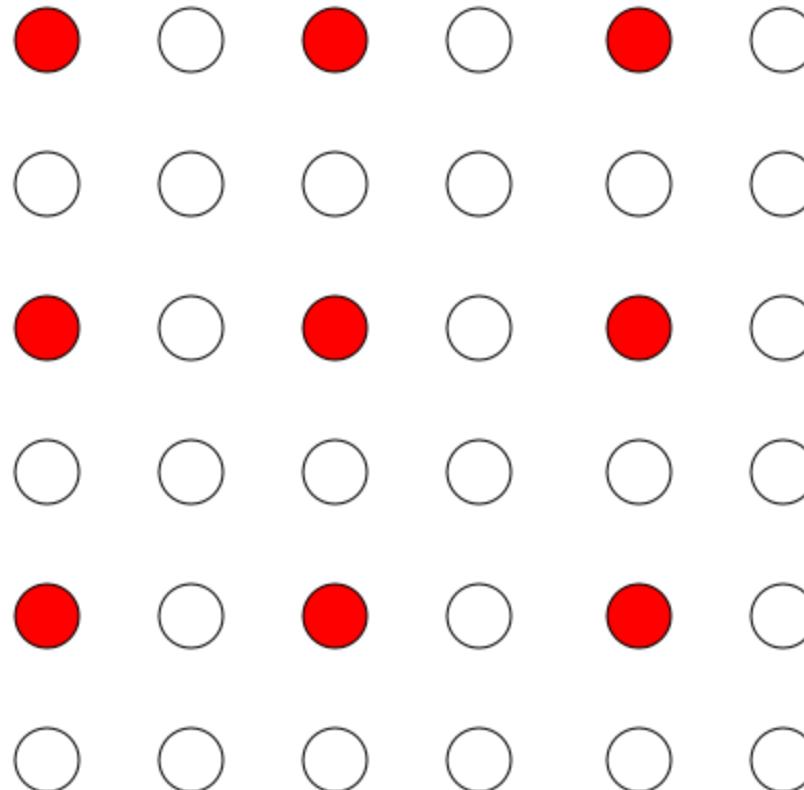
Low-Res.

High-Res.

GOOD IMAGES
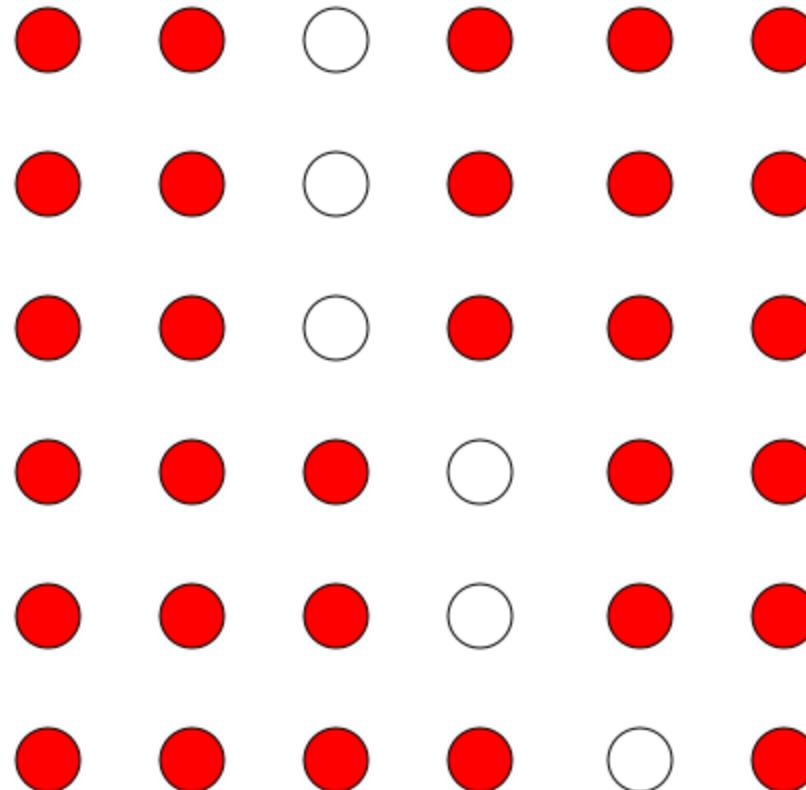
🔴 **Non-damaged**   ⚪ **Damaged**

COOL IMAGES
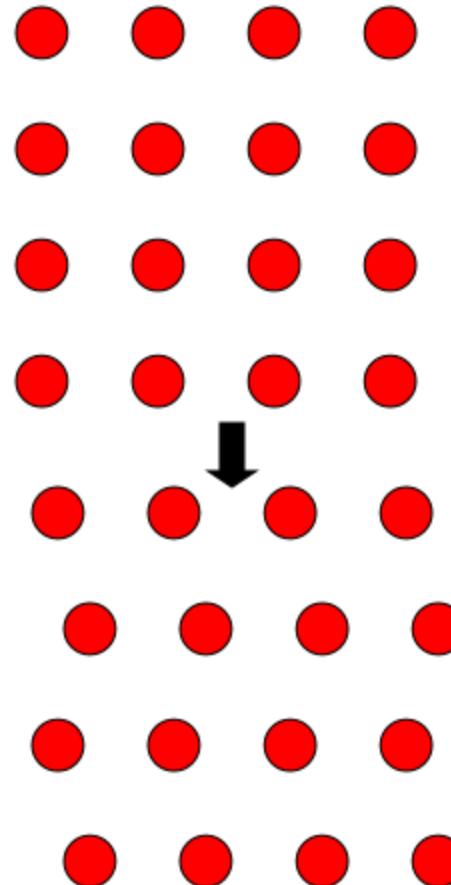
最近邻插值（Pixel Replication）

1维信号

$f(x)$

$f'(x)$
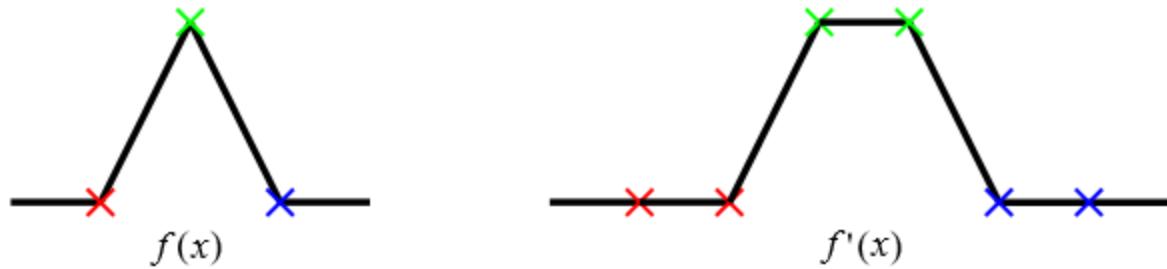
2维图像
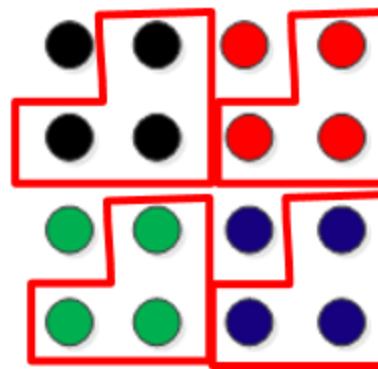
input

output

最近邻插值

Input image 256×256

Output high-resolution image 512×512

最近邻插值

**low-resolution image (100×100)**
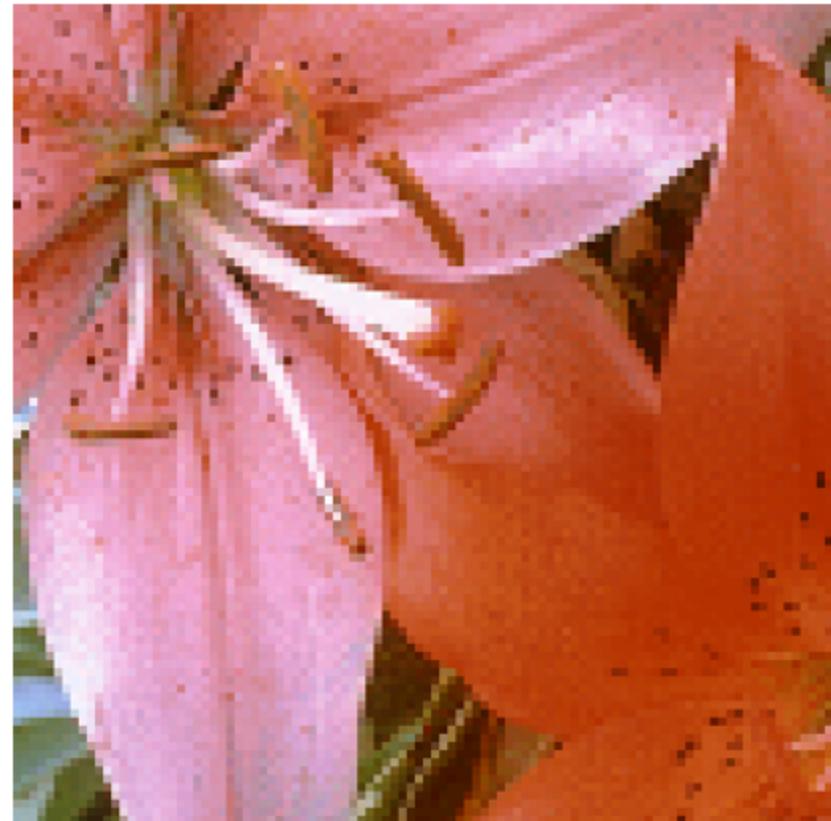
high-resolution image (400×400)

线性插值(Linear interpolation)

- **线性插值**

$$f(x_0 + d) = f(x_1) \cdot d + f(x_0) \cdot (1 - d)$$
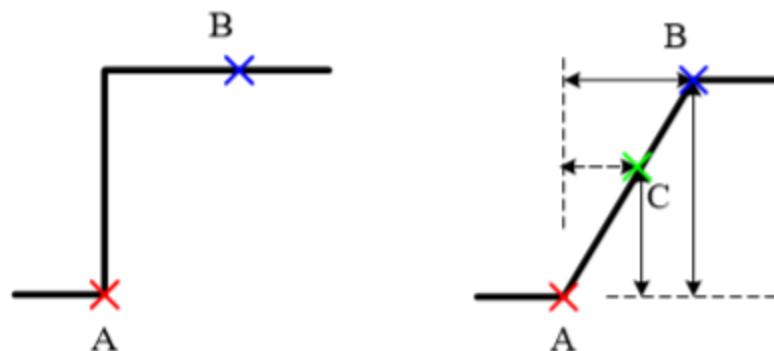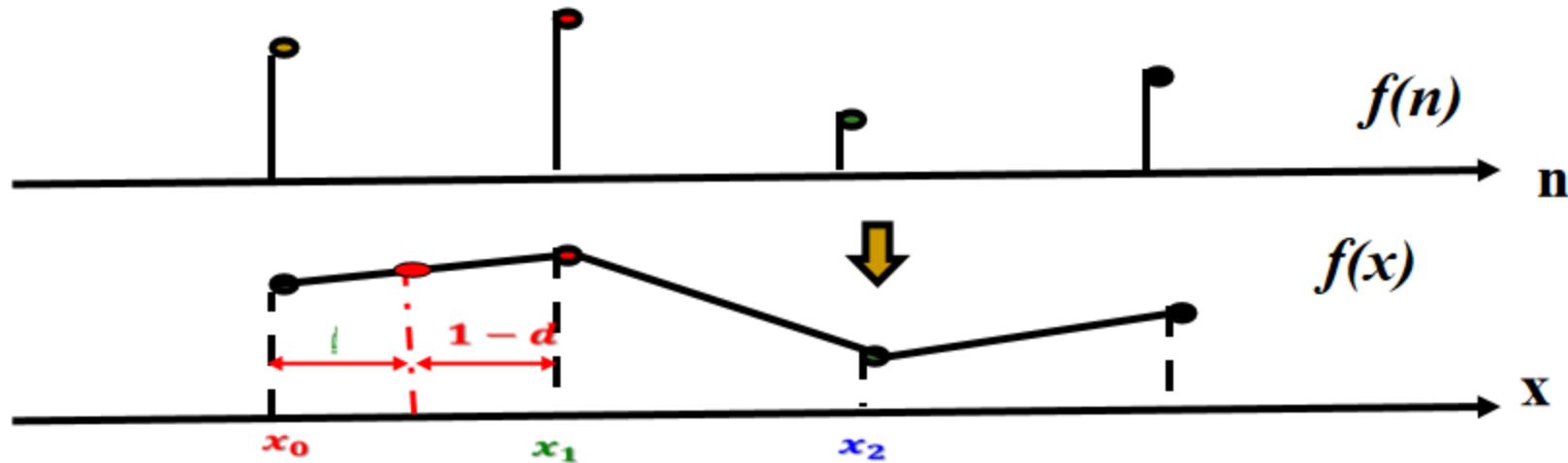
双线性插值
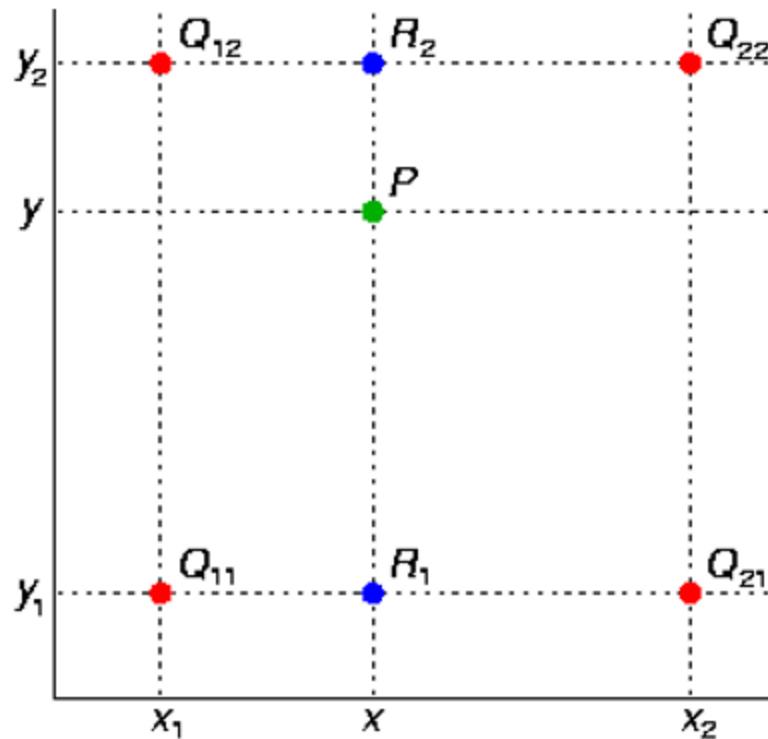
首先在 x 方向进行线性插值，得到

$$f(R_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21})$$

$$f(R_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22})$$

$$R_1 = (x, y_1), \quad R_2 = (x, y_2).$$

然后在 y 方向进行线性插值，得到

$$f(P) \approx \frac{y_2 - y}{y_2 - y_1} f(R_1) + \frac{y - y_1}{y_2 - y_1} f(R_2).$$

$$f(x, y) \approx \frac{f(Q_{11})}{(x_2 - x_1)(y_2 - y_1)}(x_2 - x)(y_2 - y) + \frac{f(Q_{21})}{(x_2 - x_1)(y_2 - y_1)}(x - x_1)(y_2 - y)$$

$$+ \frac{f(Q_{12})}{(x_2 - x_1)(y_2 - y_1)}(x_2 - x)(y - y_1) + \frac{f(Q_{22})}{(x_2 - x_1)(y_2 - y_1)}(x - x_1)(y - y_1).$$

$$f(x, y) \approx ax + by + cxy + d$$

双线性插值

Input image 256×256

Output high-resolution image 512×512

双线性插值
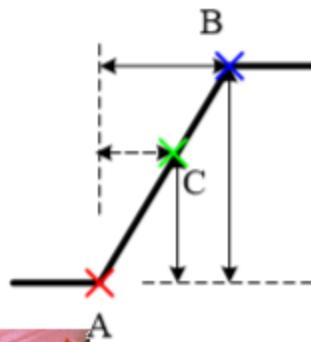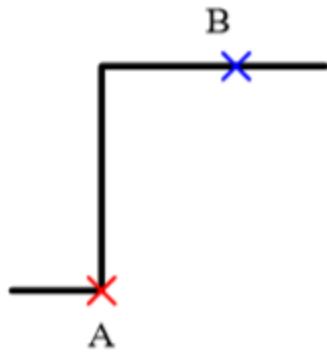
low-resolution
image (100×100)
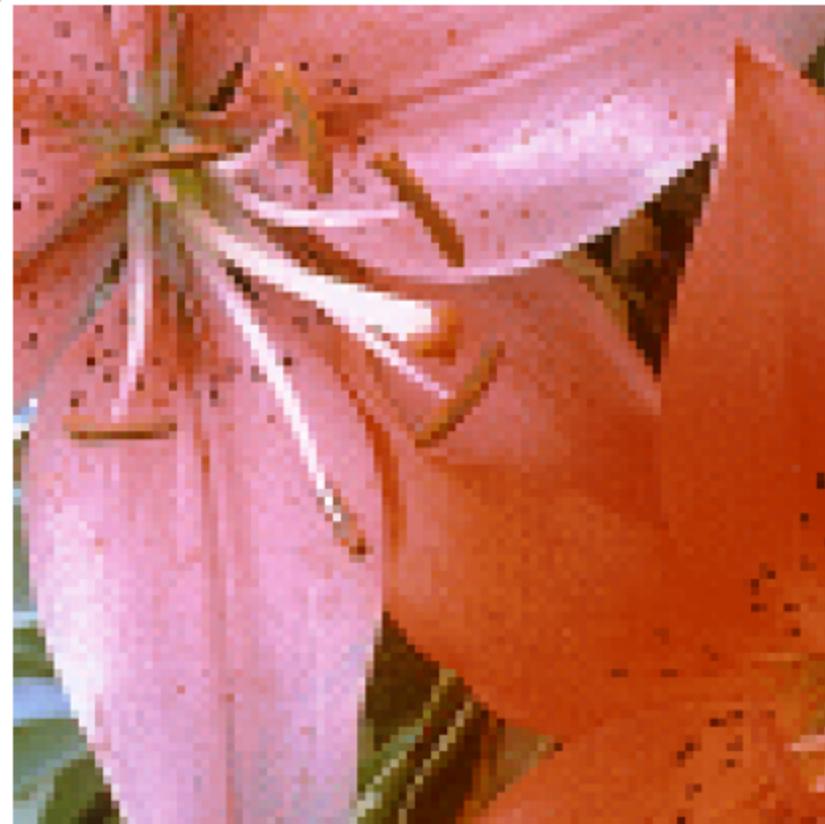
high-resolution
image (400×400)

双三次插值(Bicubic interpolation)

Point to estimate (i',j')

Final image

(i,j)

dx

dy

Extact transformed
position of (i',j')

Original image
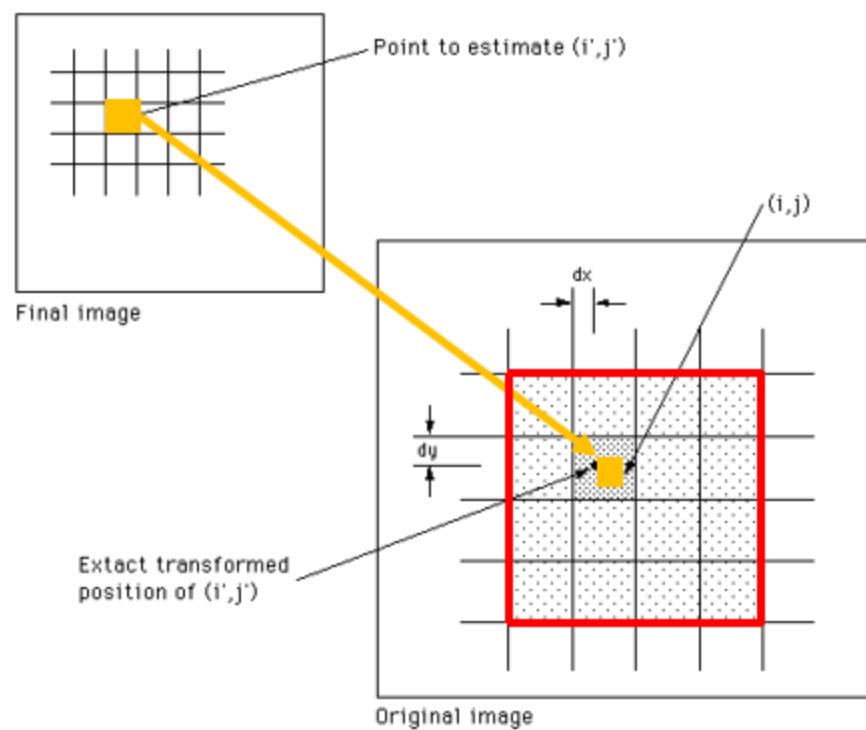
$$g(x,y) = \sum_{i=-1}^{2} \sum_{j=-1}^{2} f_{ij} W(i) W(j)$$

$$W(x) = \begin{cases} (a+2)|x|^3 - (a+3)|x|^2 + 1 & |x| \le 1 \\ a|x|^3 - 5|x|^2 + 8a|x| - 4a & 1 < |x| < 2 \\ 0 & otherwise \end{cases}$$

BiCubic函数具有如下形状：

-2.0   -1.0   0.0   1.0   2.0
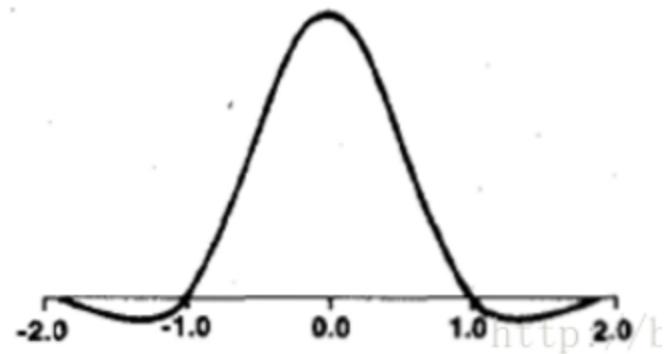
1. ^ R. Keys (1981). "Cubic convolution interpolation for digital image processing". *IEEE Transactions on Acoustics, Speech, and Signal Processing*. **29** (6): 1153–1160. doi:10.1109/TASSP.1981.1163711 .

双三次插值

Input image 256×256

Output high-resolution image 512×512

双三次插值

low-resolution
image (100×100)

high-resolution
image (400×400)