3. Intensity transformations & spatial filtering

- Background
- Some basic intensity transformation
- Histogram-based image processing
- Fundamentals of spatial filtering
- Smoothing spatial filters
- Sharpen spatial filters

3.3 Histogram processing

- **Motivation**

Intensity transform can enhance image because it properly changes the image histogram. So we can directly design an intensity transform function based on histogram

3.3 Histogram processing

➢ Definition of histogram (hist=bar,gram=图)

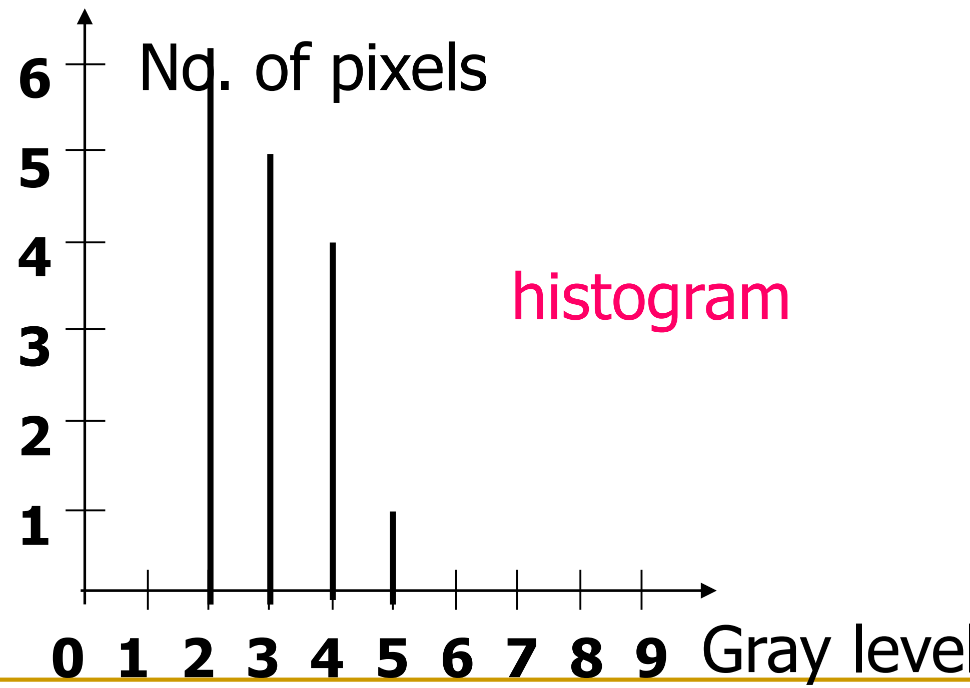
If the intensity levels is in the range [0,L-1], the histogram is a discrete function $h(r_k)=n_k$, (k=0,1,...,L-1).

where r_k is the kth intensity value, and n_k is the number of pixels in the image with intensity r_k.

2	3	3	2
4	2	4	3
3	2	3	5
2	4	2	4

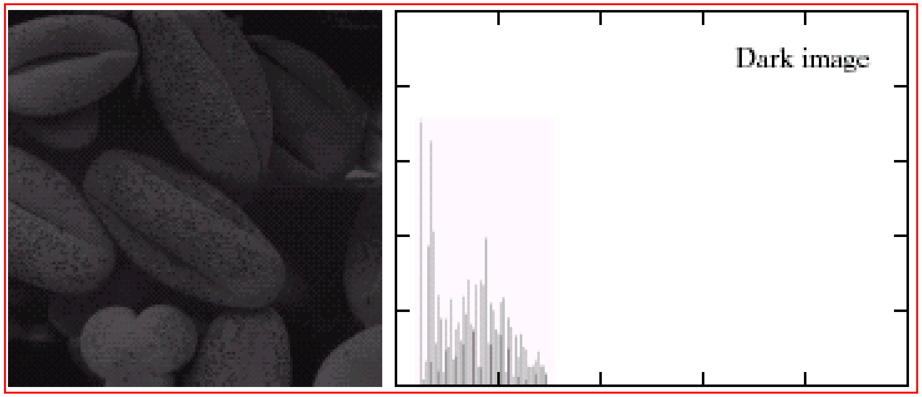
4x4 image

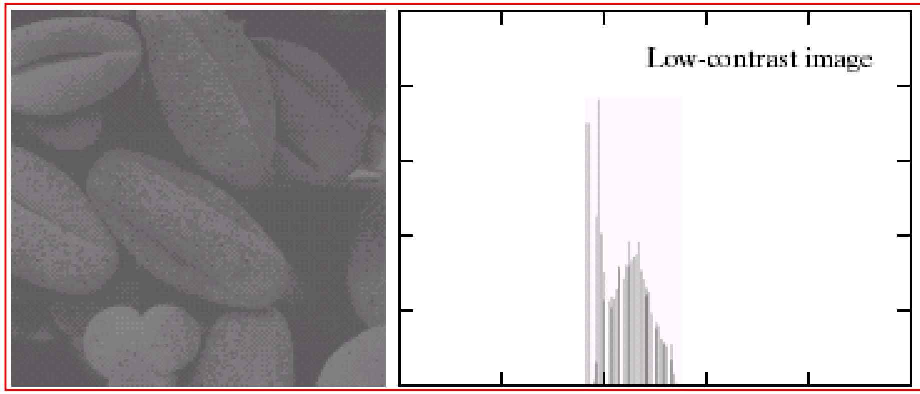
Gray scale = [0,9]

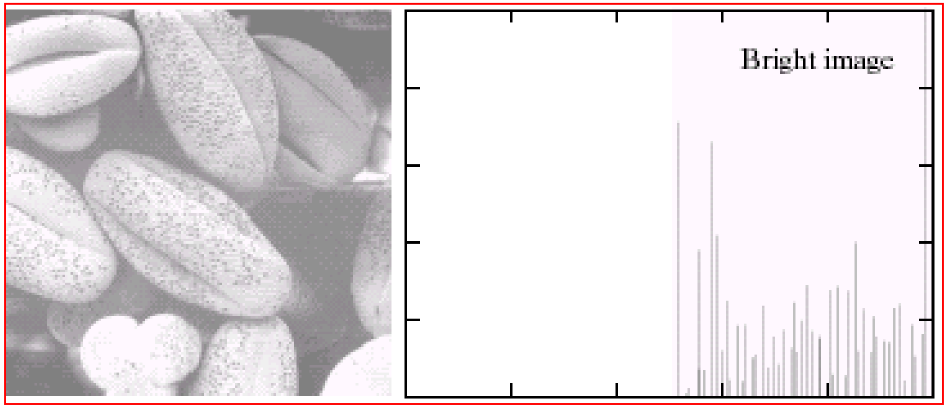


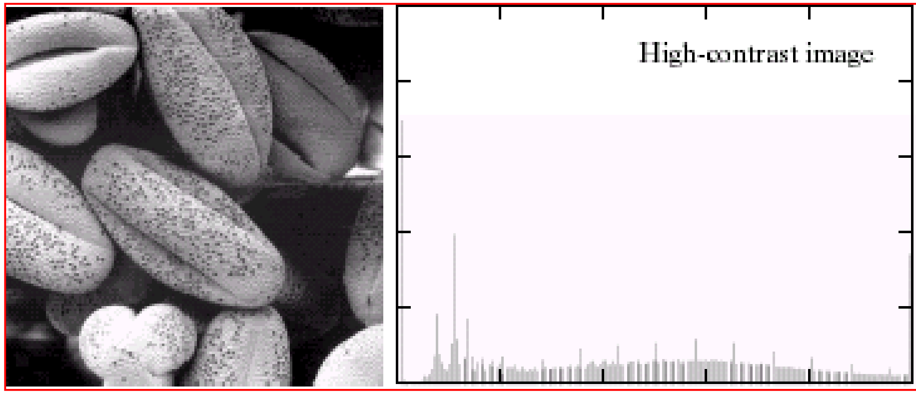
3.3 Histogram processing

➢ Motivation









3.3 Histogram processing

histogram processing

- histogram equalization (HE)
- histogram specification (HS)
- Local HE

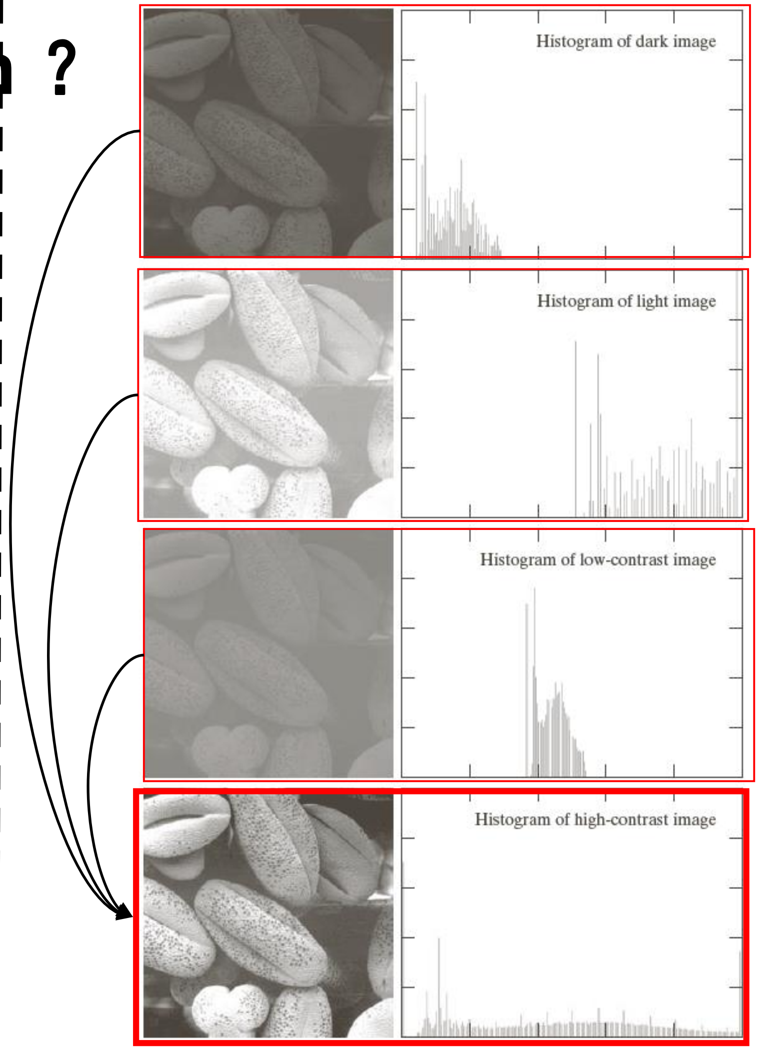
3.3.1 Histogram Equalization (HE)

➢ What's histogram equalization ?

A process that seek an intensity transform function $s=T(r)$ so that the histogram of transformed image becomes flat.

The advantages of HE

Automatically and adaptly determine an optimal transform function

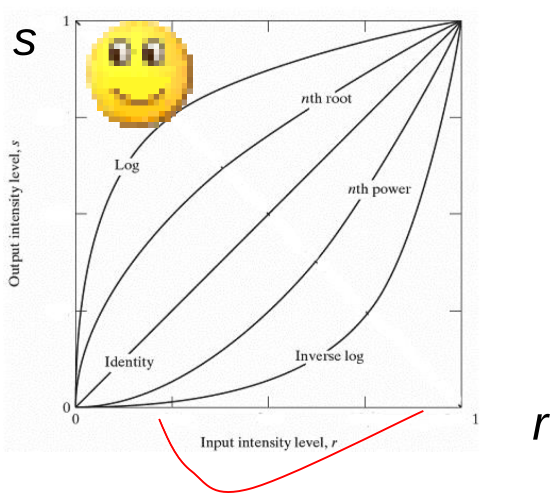


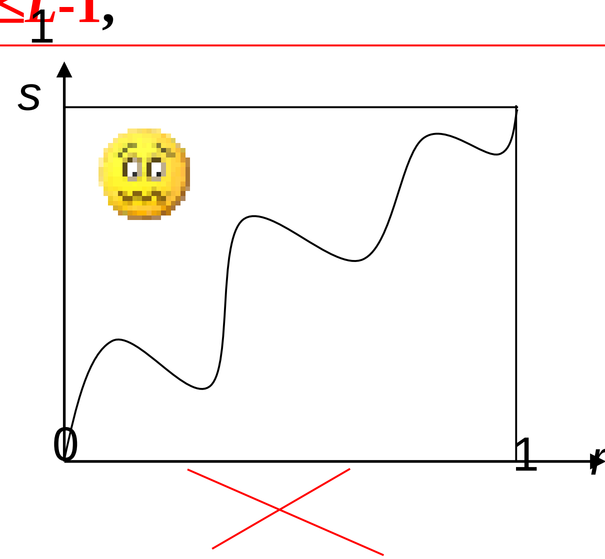
3.3.1 Histogram Equalization (HE)

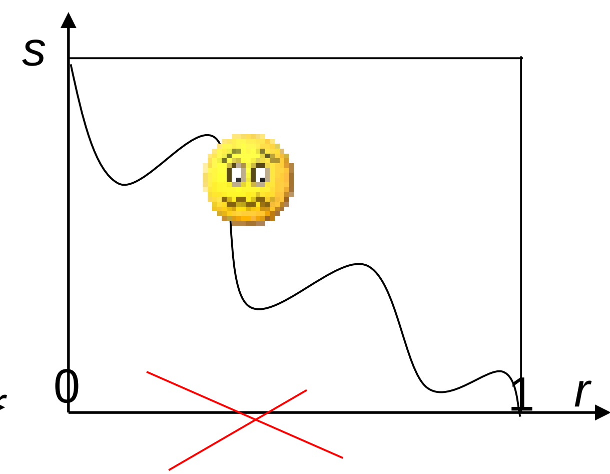
➢ The constraints of HE

Let $r \in [0, L-1]$ be the input intensity, $s = T(r)$ is the transformed intensity. It is required that the function T satisfies:

1) **$T(r)$ is a monotonically increasing function in the interval $0 \le r \le L-1$.**
2) **$0 \le T(r) \le L-1$ for $0 \le r \le L-1$,**







3.3.1 Histogram Equalization (HE)

How to HE based on the relationship

The formula of HE

If $s = T(r) = (L-1)\int_0^r p_{r(w)dw}$, then $p_s(s) = \frac{1}{L-1}$,0≤s ≤L-1

Cumulative distribution function (CDF) is used as the HE transform function

Proof: Because $$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

so $$\frac{dT(r)}{dr} = \frac{d[(L-1)\int_0^r p_r(w)dw]}{dr} = (L-1)p_r(r)$$

substitute $$p_s(s) = p_r(r)\frac{1}{dT(r)/dr} = p_r(r) * \frac{1}{(L-1)p_r(r)} = \frac{1}{L-1}$$

随机变量s具有均匀PDF表征

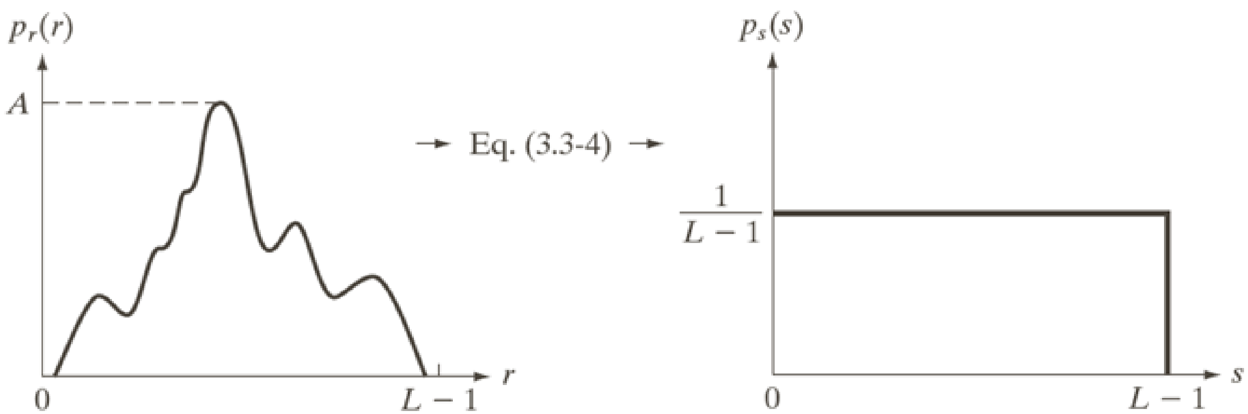
3.3.1 Histogram Equalization (HE)

If

The formula of HE

If $s = T(r) = (L-1)\int_0^r p_r(w)\,dw$, then $p_s(s) = \frac{1}{L-1}$, $0 \le s \le L-1$

Cumulative distribution function (CDF) is used as the HE transform function



a b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, *r*. The resulting intensities, *s*, have a uniform PDF, independently of the form of the PDF of the *r*'s.

3.3.1 Histogram Equalization (HE)

The formula of HE

If $s = T(r) = (L-1)\int_0^r p_{r(w)dw}$,then $p_s(s) = \frac{1}{L-1}$,0≤s ≤L-1

discrete

$$p_k(r_k) = \frac{n_k}{n} \quad k = 0,1,\ldots,L-1$$

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)\sum_{j=0}^{k}\frac{n_j}{n} \quad k = 0,1,\ldots,L-1$$

例 1 . 直方图均衡化

(1) 统计原始图像的直方图

对64×64的图像, L=8, 图像中各灰度级的像素数目为:

r_k（灰度级）	n_k	r_k（灰度级）	n_k
0	790	4	329
1	1023	5	245
2	850	6	122
3	656	7	81

序号	运　算	步骤和结果							
1	原始图像灰度级 r_k	0	1	2	3	4	5	6	7
2	原始直方图 $p_r(r_k)=\frac{n_k}{n}$	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02

(2) 计算直方图累积分布函数

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n}$$

序号	运 算	步骤和结果							
1	原始图像灰度级 r_k	0	1	2	3	4	5	6	7
2	原始直方图 $p_r(r_k) = \frac{n_k}{n}$	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02
3	计算累积直方图各项 $s_k = \sum_{j=0}^{k} \frac{n_j}{n}$	0.19	0.44	0.65	0.81	0.89	0.95	0.98	1.00

(3) 用累积分布函数作变换函数进行图像灰度变换

$$S(k) = \text{int}\left[((L-1)-0)\cdot s_k + 0.5\right]$$

序号	运　算	步骤和结果							
1	原始图像灰度级 r_k	0	1	2	3	4	5	6	7
2	原始直方图 $p_r(r_k)=\frac{n_k}{n}$	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02
3	计算累积直方图各项 $s_k=\sum_{j=0}^{k}\frac{n_j}{n}$	0.19	0.44	0.65	0.81	0.89	0.95	0.98	1.00
4	取整扩展 $S(k)=\text{int}\left[((L-1)-0)\cdot s_k+0.5\right]$	1	3	5	6	6	7	7	7

例 直方图均衡化

序号	运算	步骤和结果							
1	原始图像灰度级 r_k	0	1	2	3	4	5	6	7
2	原始直方图 $p_r(r_k)=\frac{n_k}{n}$	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02
3	计算累积直方图各项 $s_k=\sum_{j=0}^{k}\frac{n_j}{n}$	0.19	0.44	0.65	0.81	0.89	0.95	0.98	1.00
4	取整扩展: $S(k)=\text{int}\left[\left((L-1)-0\right)\cdot s_k+0.5\right]$	1	3	5	6	6	7	7	7
5	确定映射对应关系 $r_k\rightarrow S(k)$	0→1	1→3	2→5	3,4→6		5,6,7→7		
6	根据映射关系计算均衡化直方图		0.19		0.25		0.21	0.24	0.11

例

均衡化前后直方图比较

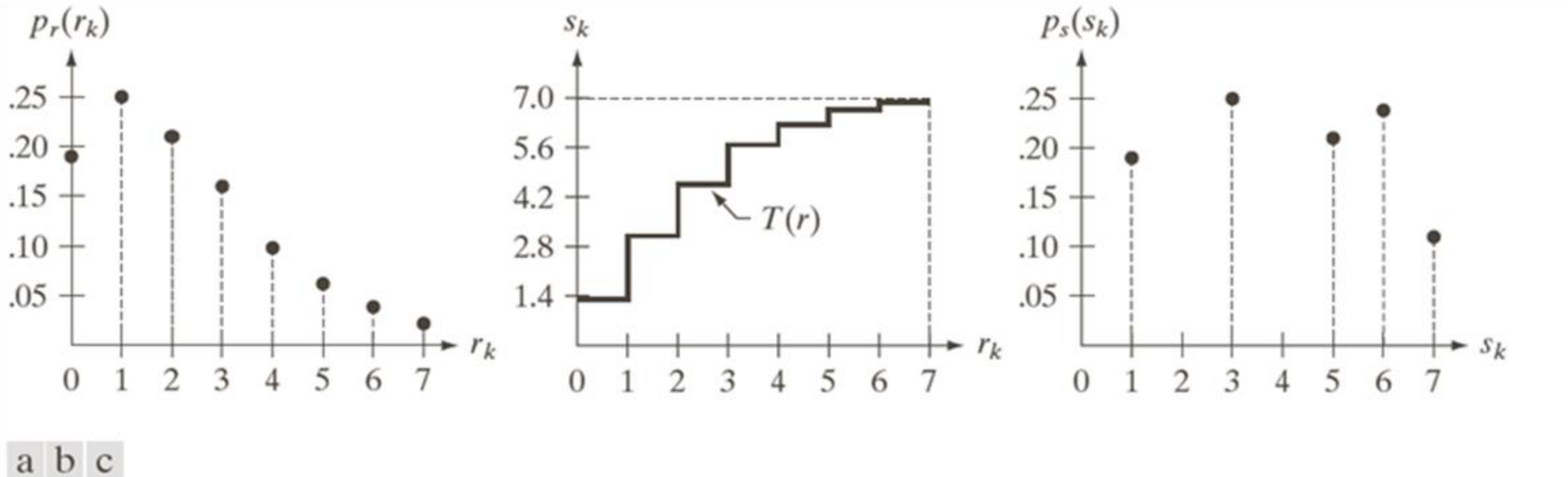


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

小结：

（1）因为直方图是概率密度函数的近似，而且均衡化过程中不产生新的灰度级，所以直方图均衡化很少得到完全平坦的结果；

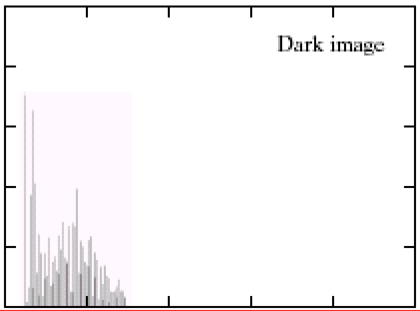
（2） 变换后灰度级减少，即出现灰度“简并”现象，造成一些灰度层次的损失。

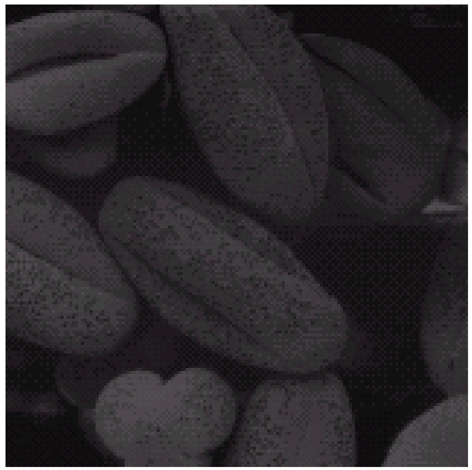
3.3.1 Histogram Equalization (HE)

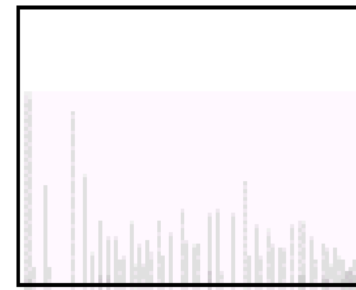
➢Examples

HE

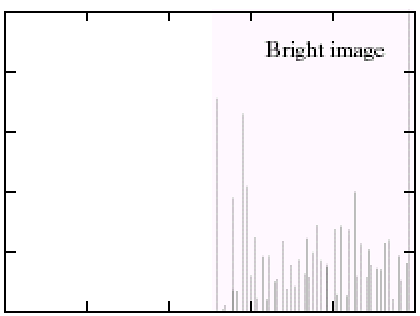
暗图像







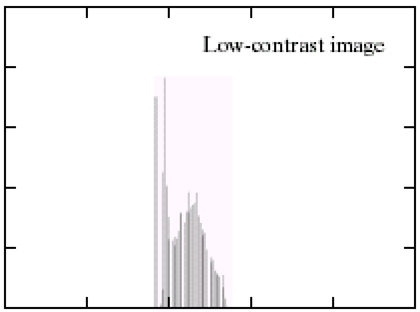
亮图像

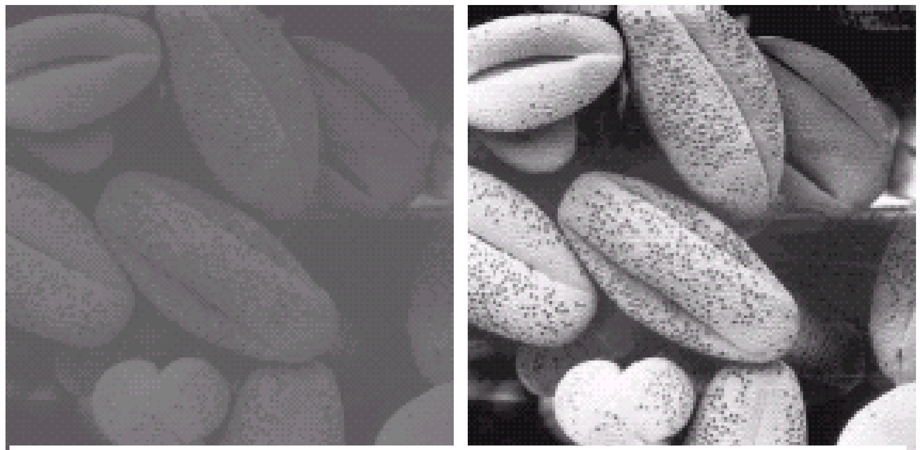


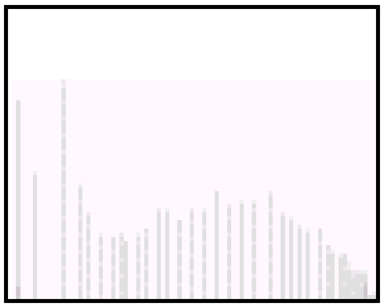
3.3.1 Histogram Equalization (HE)

- **Examples**

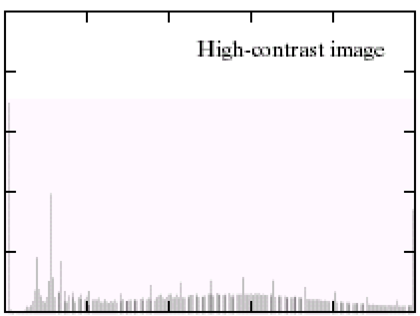
低对比度图像

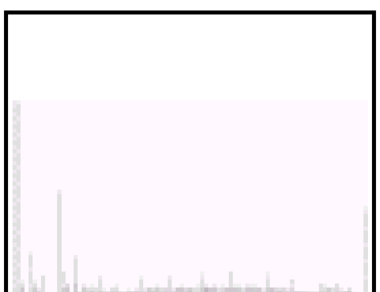






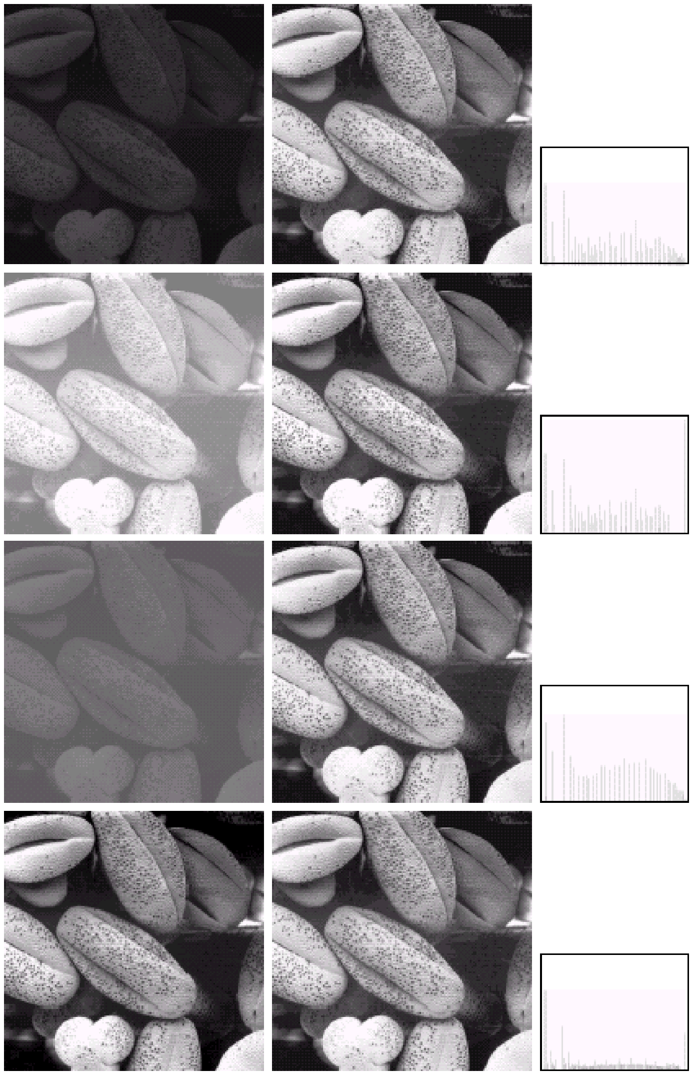
高对比度图像



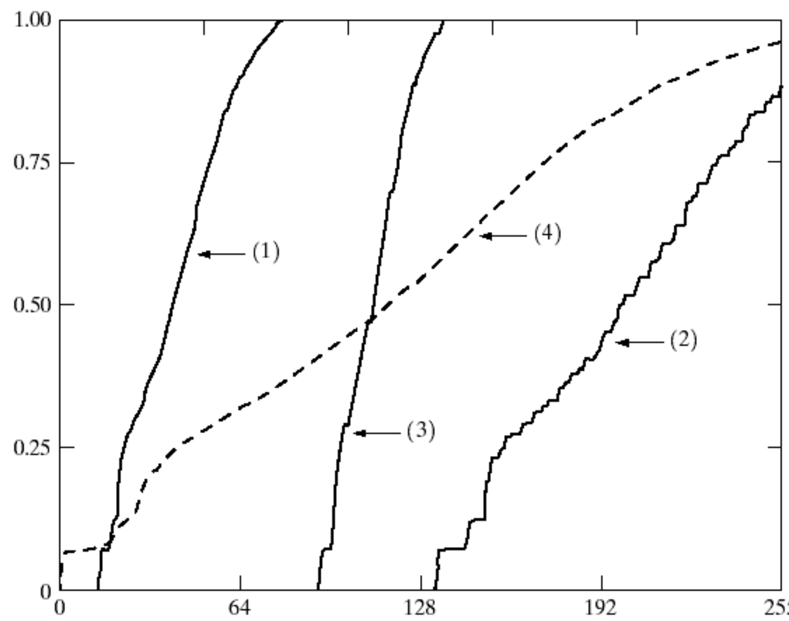


3.3.1 Histogram Equalization (HE)

➢Examples

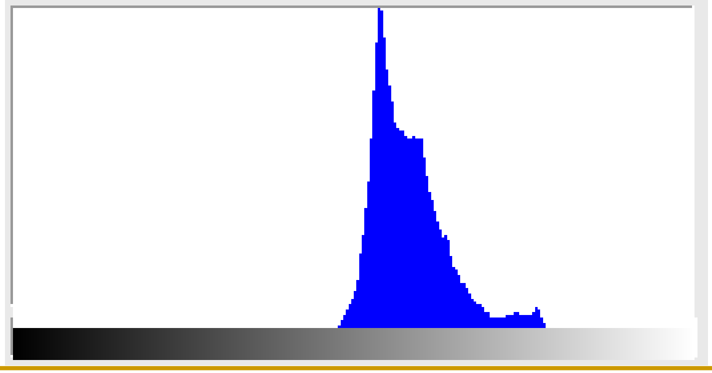


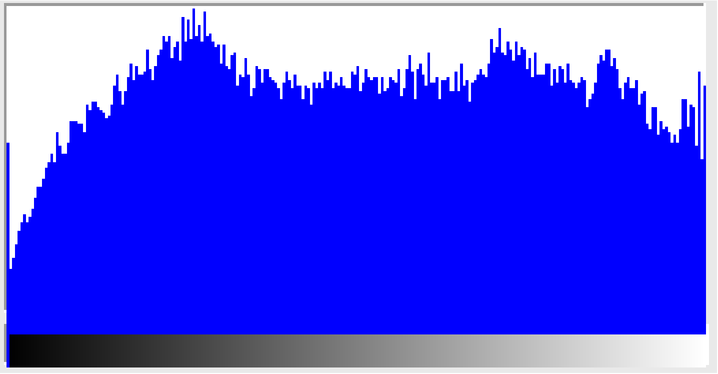
- 原始图像差别很大, 经直方图均衡化后的图像视觉效果接近
- 高对比度图像经直方图均衡化后变化不大



3.3.1 Histogram Equalization (HE)

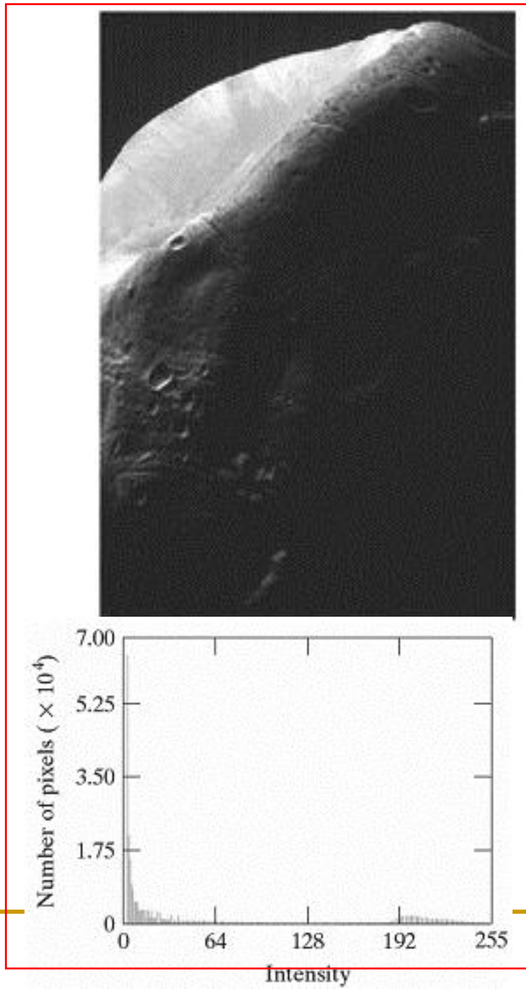
➢ Examples



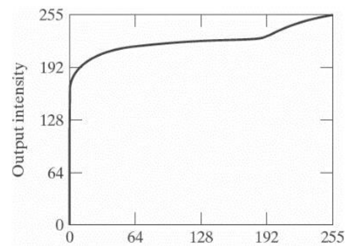


3.3.2 Histogram specification (matching)

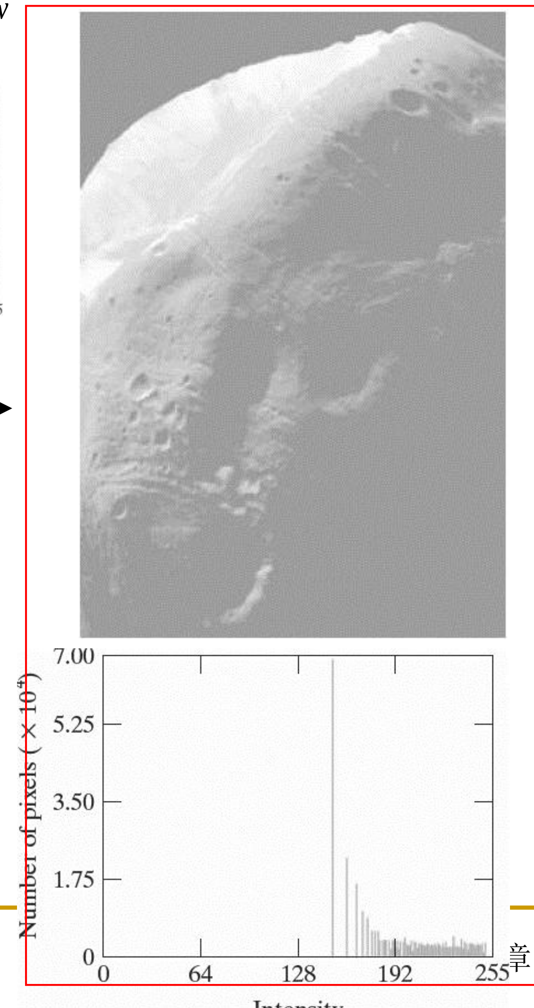
- **Motivation: overcome the shortcoming of HE**



$$s = T(r) = \int_0^r p_r(w)dw$$



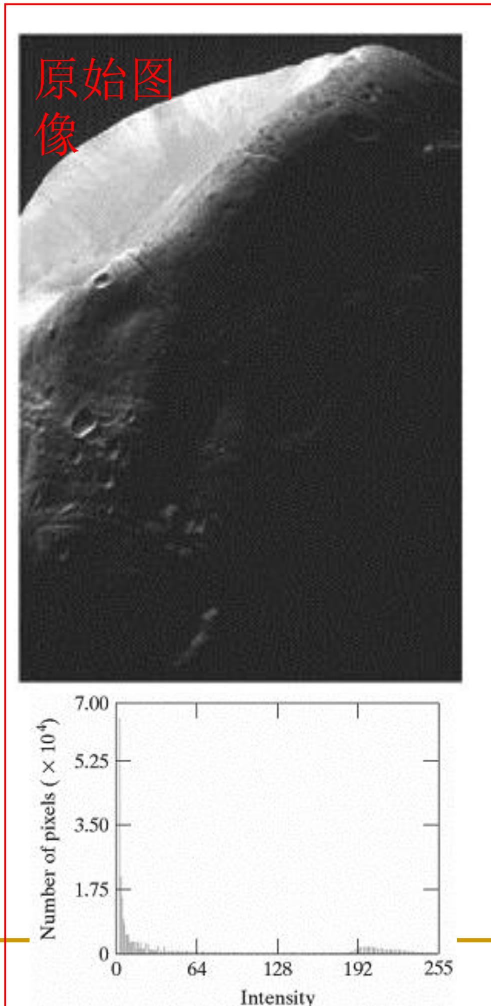
Light, washed out appearance

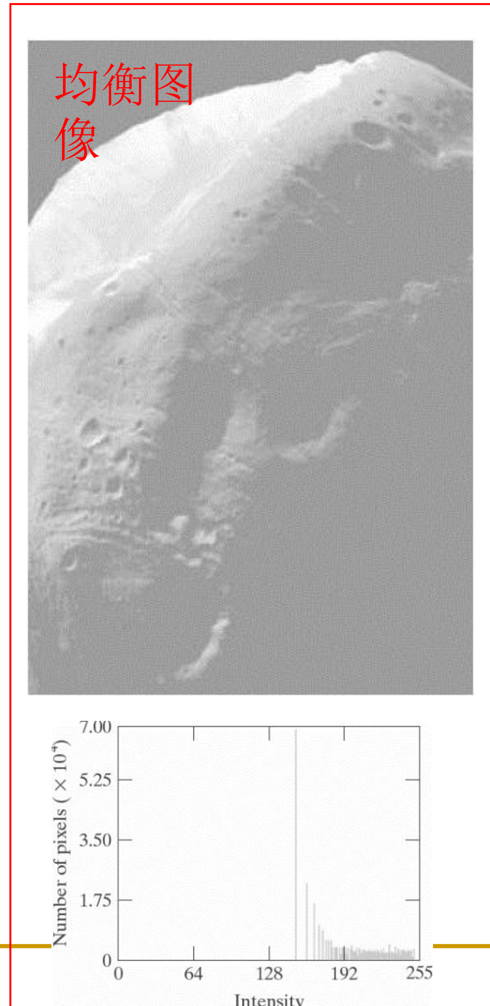


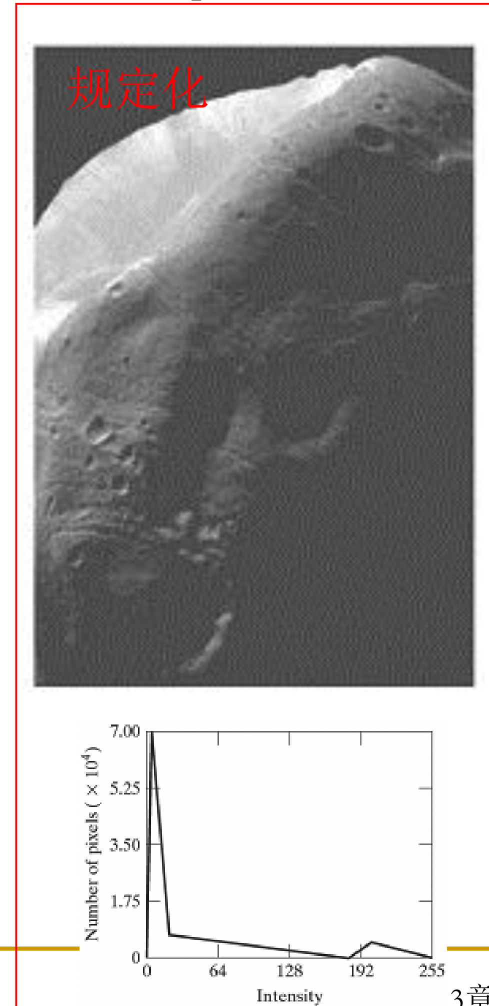
3.3.2 Histogram specification (matching)

- **Motivation: overcome the shortcoming of HE**

Specify a possible and reasonable output histogram







3.3.2 Histogram specification (matching)

➢The principle of histogram specification

Given: 输入图像的直方图为$p_r(r)$， 规定输出图像的直方图为 $p_z(z)$

Find: 一个变换 $r \mapsto z$ ，使得输出图像的直方图为$p_z(z)$

考察使输入图像的直方图均衡化的变换函数：$T(r): r \mapsto s$

$$s = T(r) = (L-1)\int_0^r p_{r(w)dw}$$

考察使输出图像的直方图均衡化的变换函数：$G(z): z \mapsto s$

$$G(z) = (L-1)\int_0^z p_{z(t)dt} = s$$

得到： $z = G^{-1}(s) = G^{-1}[T(r)]$

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) \qquad z_k = G^{-1}(s_k) = G^{-1}(T(r_k))$$

3.3.2 Histogram specification (matching)

直方图规定化的实现

(1)求出已知图像的直方图$P_r(r)$

(2)寻找 $P_r(r)$ 的直方图均衡变换，对每一灰度级r_k预计算映射灰度级s_k.

(3)利用 $v_k = G(z_k) = \sum_{i=0}^{k} p_z(z_i)$ **从给定的P_z(z)得到变换函数G.**

(4) 对一个s_k值计算满足 $G(z_k) - s_k = 0$ 的最接近整数z_k

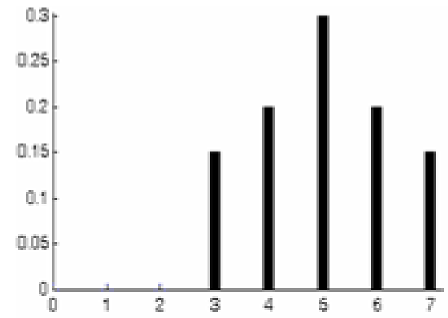
(5) 对于原始图像的每个像素,若像素值为r_k,将该值映射到其对应的灰度级s_k;然后映射灰度级s_k到最终灰度级z_k.

例：P 8 0

$$r_j \rightarrow s_j$$

序号	运　算	步骤和结果							
1	原始图像灰度级 r_k	0	1	2	3	4	5	6	7
2	原始直方图 $p_r(r_k)=\dfrac{n_k}{n}$	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02
3	计算累积直方图各项 $s_k=\sum_{j=0}^{k}\dfrac{n_j}{n}$	0.19	0.44	0.65	0.81	0.89	0.95	0.98	1.00
4	取整扩展: $S(k)=\text{int}\left[\left((L-1)-0\right)\cdot s_k+0.5\right]$	1	3	5	6	6	7	7	7
5	确定映射对应关系 $r_k \rightarrow S(k)$	0→1	1→3	2→5	3,4→6		5,6,7→7		
6	根据映射关系计算均衡化直方图		0.19		0.25		0.21	0.24	0.11

$z_k \leftrightarrow v_k$



规定直方图

序列	运算	步骤和结果							
1	期望图像灰度级 z_k	0	1	2	3	4	5	6	7
2	期望直方图 $p_z(z_k)$	0.0	0.0	0.0	0.15	0.20	0.30	0.20	0.15
3	计算累计直方图各项 $G(z_k)$	0.0	0.0	0.0	0.15	0.35	0.65	0.85	1
4	取整扩展 $\mathrm{G}(z_k)=\mathrm{int}\left[\left((L-1)-0\right)\cdot \mathrm{G}(z_k)+0.5\right]$	0	0	0	1	3	5	6	7
5	确定映射对应关系 $z_k \rightarrow G(z_k)$	0−>0	1−>0	2−>0	3−>1	4−>3	5−>5	6−>6	7−>7

$$p_z(z)$$

↓ 均衡

$$r_j \xrightarrow{\text{均衡}} s_j \approx G(z_k) \xrightarrow{G^{-1}} z_k$$

r_k	→	s_k
0	→	1
1	→	3
2	→	5
3、4	→	6
5、6、7	→	7

$G(z_k)$	→	z_k
0	→	1
1	→	3
3	→	4
5	→	5
6	→	6
7	→	7

所以最后结果：

$r_0 \rightarrow z_3$

$r_1 \rightarrow z_4$

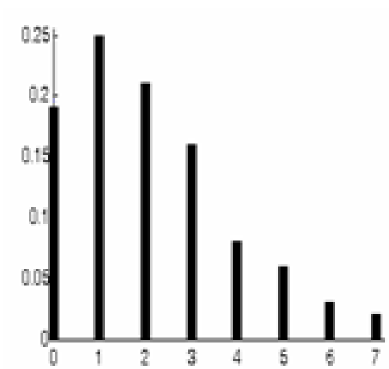
$r_2 \rightarrow z_5$

r_3、$r_4 \rightarrow z_6$

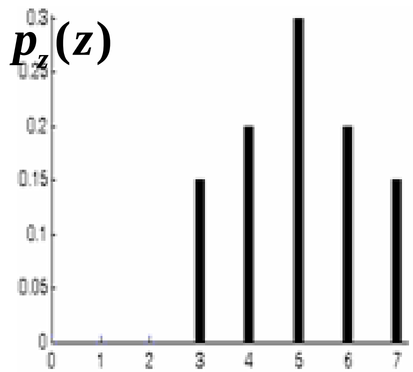
r_5、r_6、$r_7 \rightarrow z_7$

规定化后的直方图 $\tilde{p}_z(z)$

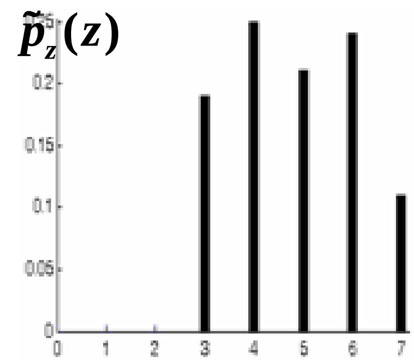
灰度级	0	1	2	3	4	5	6	7
像素	0	0	0	790	1023	850	985	448
概率	0.00	0.00	0.00	0.19	0.25	0.21	0.24 0.16＋0.08	0.11 0.06＋0.03＋0.02



原始直方图



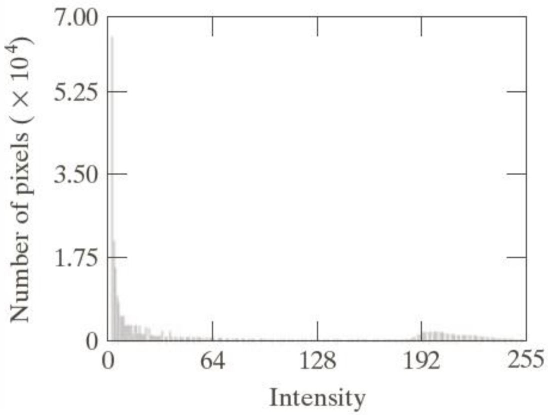
规定直方图

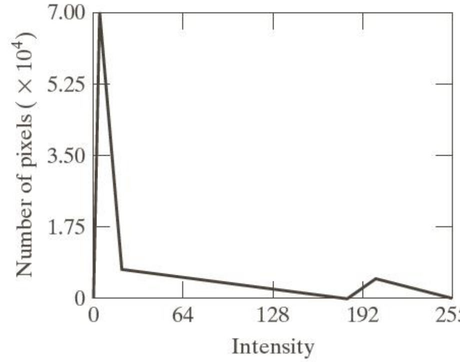


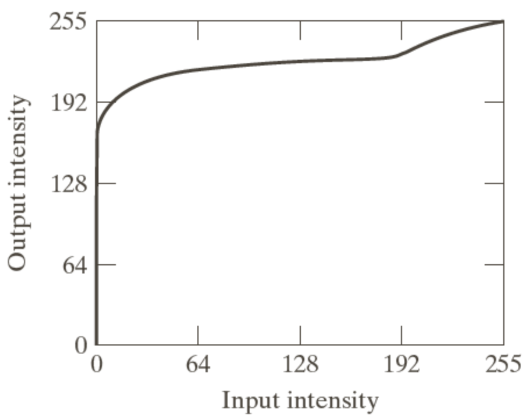
结果直方图

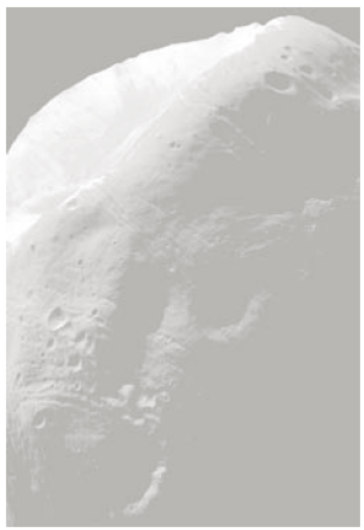
原始图像

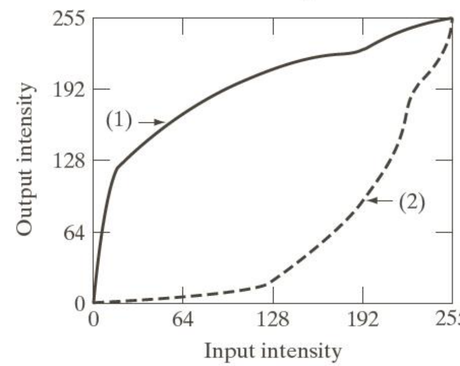
P 8 2　例3.9

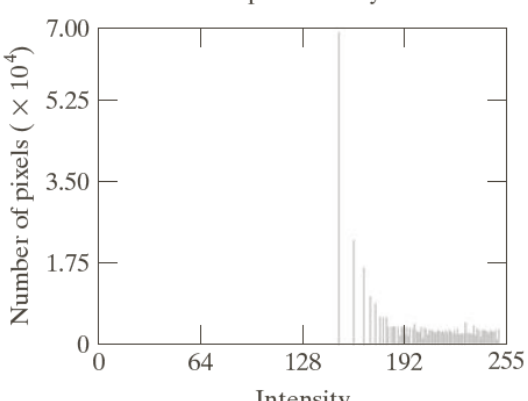




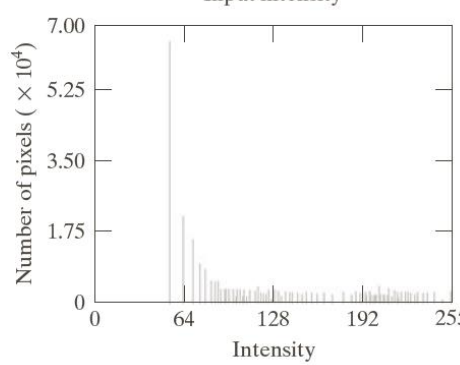








直方图均衡处理



直方图规定化处理