3. Intensity transformations & spatial filtering

- Background
- Some basic intensity transformation
- Histogram-based image enhancement
- Fundamentals of spatial filtering
- Smoothing spatial filters
- Sharpen spatial filters

3.4 Fundamentals of spatial filtering

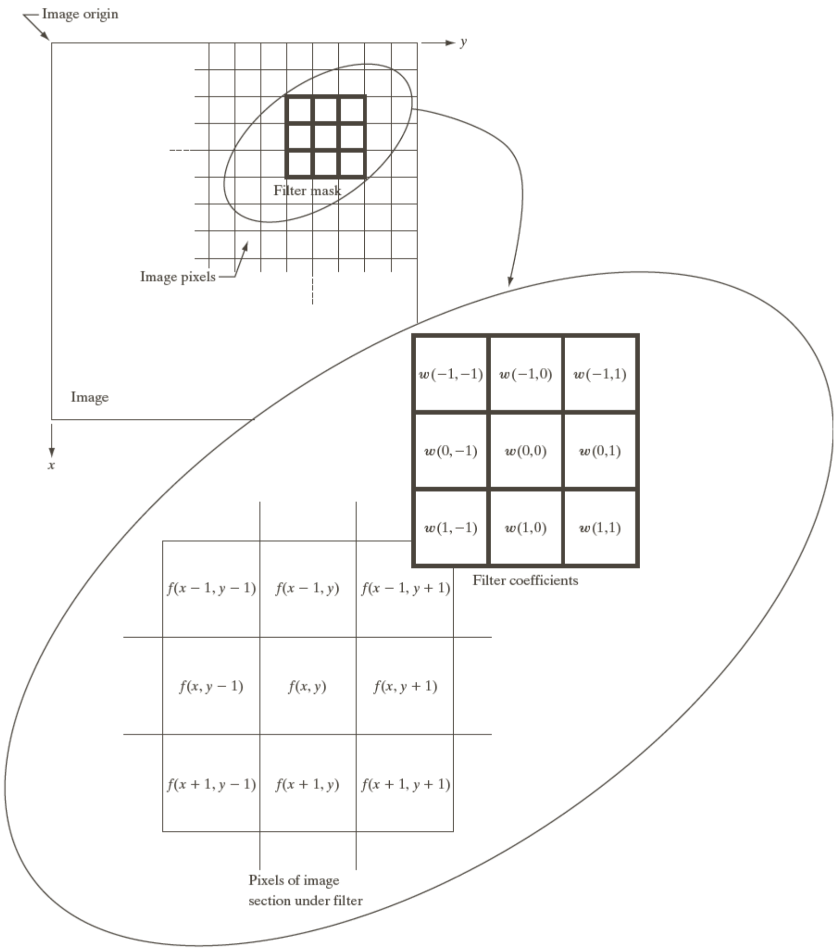
➢Spatial filters vs. frequency filters

1) one-to-one correspondence in many cases

2) Spatial filters offer considerably more versatility because they can be used also for nonlinear filtering, something we cannot do in the frequency domain

3.4 Fundamentals of spatial filtering

Filter（滤波器）
Mask（掩模）
Kernel（核）
Template（模板）
Window（窗口）



$$g(x,y) = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \ldots + w(0,0)f(x,y) + \ldots + w(1,0)f(x+1,y) + w(1,1)f(x+1,y+1)$$

3.4 Fundamentals of spatial filtering

- The general form of linear filtering

给定输入图像$f(x,y)$，和尺寸为$m\times n$的模板$w(i,j)$， 则滤波后的输出图像$g(x,y)$为：

$$g(x,y)=\sum_{s=-a}^{s=a}\sum_{t=-b}^{b} w(s,t)f(x+s,y+t)$$

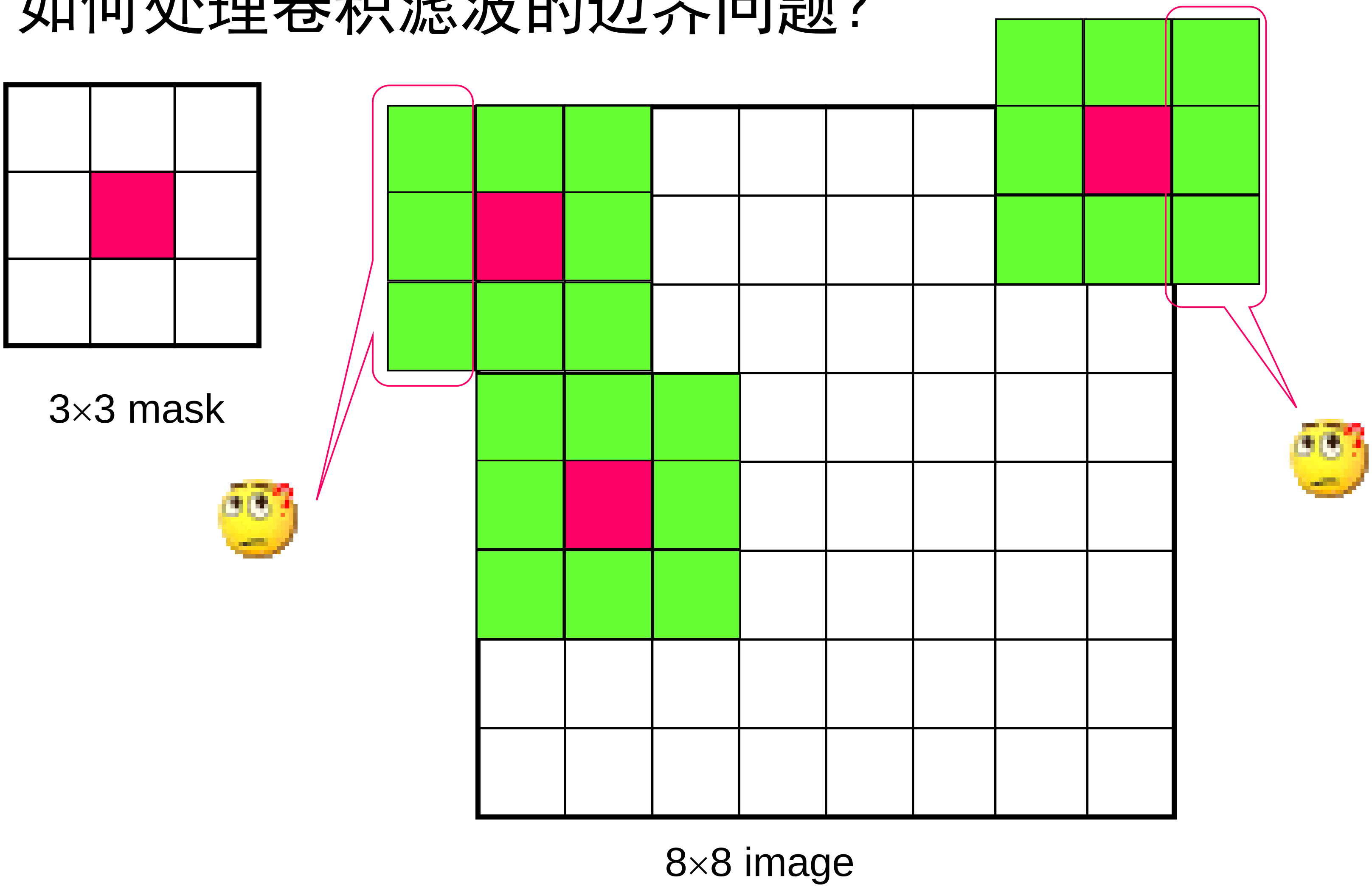
其中： $a=\frac{m-1}{2}$， $b=\frac{n-1}{2}$.

---

m和n通常取奇数，为什么？

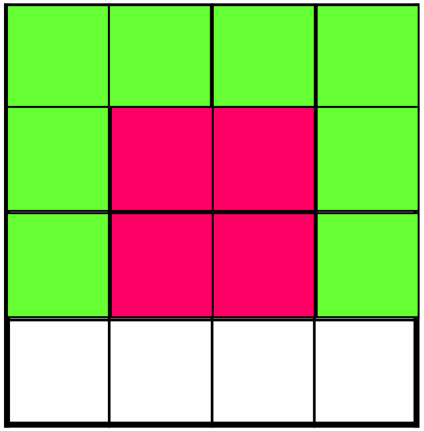
3.4 Fundamentals of spatial filtering

如何处理卷积滤波的边界问题?



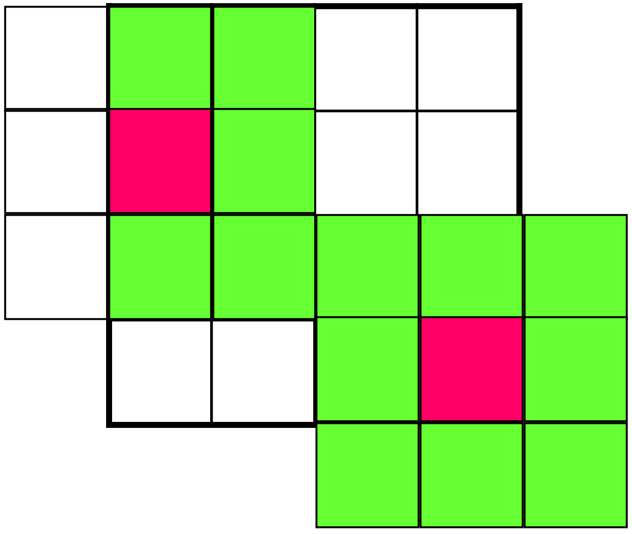
3.4 Fundamentals of spatial filtering

处理边界问题的三种方法



① 丢弃法:对边界部分不予处理, 只对距离边界大于(n-1)/2的像素滤波. 滤波后的图像比原始图像小.

② 局部法: 对边界部分用模板的重叠区域滤波, 滤波后的图像与原始图像大小相等



③ 延拓法:把图像向外延拓几个像素后, 再滤波

3.4.2 空间的相关和卷积

■ 相关和卷积的关系：

一维情况：

相关 | 卷积

Correlation

(a) Origin f: 0 0 0 1 0 0 0 0 — w: 1 2 3 2 8

(b) 0 0 0 1 0 0 0 0
1 2 3 2 8
Starting position alignment

Zero padding

(c) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8

(d) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
Position after one shift

(e) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
Position after four shifts

(f) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8
Final position

Full correlation result
(g) 0 0 0 8 2 3 2 1 0 0 0 0

Cropped correlation result
(h) 0 8 2 3 2 1 0 0

Convolution

Origin f: 0 0 0 1 0 0 0 0 — w rotated 180°: 8 2 3 2 1 (i)

0 0 0 1 0 0 0 0 (j)
8 2 3 2 1

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (k)
8 2 3 2 1

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (l)
8 2 3 2 1

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (m)
8 2 3 2 1

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (n)
8 2 3 2 1

Full convolution result
0 0 0 1 2 3 2 8 0 0 0 0 (o)

Cropped convolution result
0 1 2 3 2 8 0 0 (p)

FIGURE 3.29 Illustration of 1-D correlation and convolution of a filter with a discrete unit impulse. Note that correlation and convolution are functions of *displacement*.

3.4.2 空间的相关和卷积

- **相关和卷积的关系：**

一维情况：

$$\text{相关：}r_{xy}[n]=\sum_{k=-\infty}^{\infty}x[k]y[n+k]$$

$$\text{卷积：}x[n]*y[n]=\sum_{k=-\infty}^{\infty}x[k]y[n-k]$$

任意一个函数和冲激函数的相关相当于“复制”冲激位置上此函数的反转“版本”

为执行卷积，需先把参加运算的一个函数旋转180°，然后再执行相关中的相同操作。

3.4.2 空间的相关和卷积

相关和卷积：

二维情况：

相关：

$$(x,y) \star f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t) \quad (3.4-1)$$

卷积：

$$(x,y) \bigstar f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t) \quad (3.4-2)$$

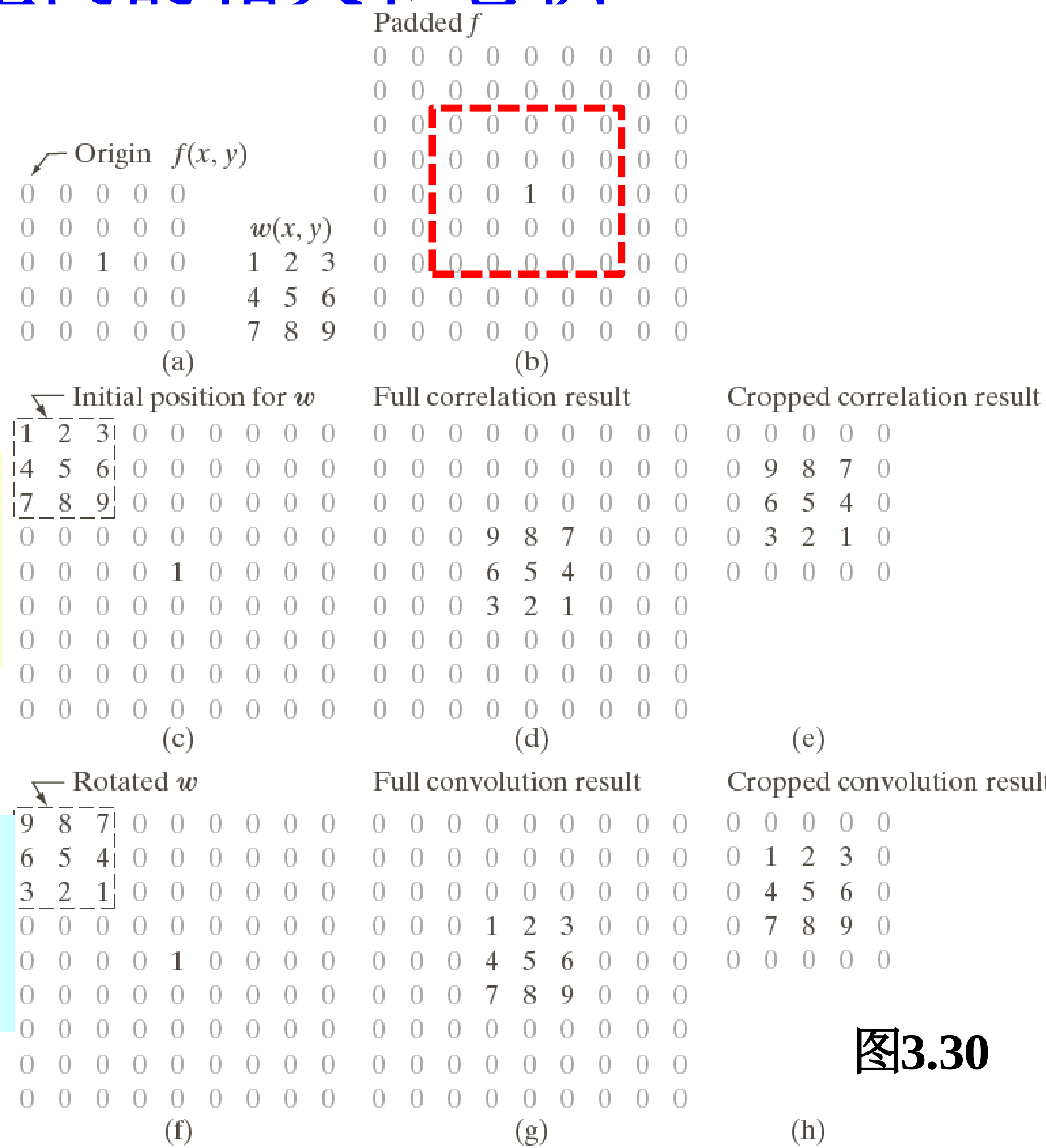


图3.30

3.5 Smoothing spatial filtering

➢The goal of smoothing spatial filtering

1) Input输入 原始图像 → 平滑滤波器(smoothing) (去噪,noise reduction) → 输出 增强图像

2) Input输入 原始图像 → 平滑滤波器(smoothing) (模糊,blurring 弱化小物体) → 阈值化 (Thresholding) → 输出 增强图像 (强化大物体)

类似图像分割

3.5.1 Smoothing linear filtering

- **Two forms: uniform weights and un-uniform weights**

1) uniform $R=\sum_{i=1}^{9}\frac{1}{9}z_i=\frac{1}{9}\sum_{i=1}^{9}z_i$

$\frac{1}{9}\times$

1	1	1
1	1	1
1	1	1

2) un-uniform $R=\sum_{i=1}^{9}w_i z_i$

Inversely proportional to distance

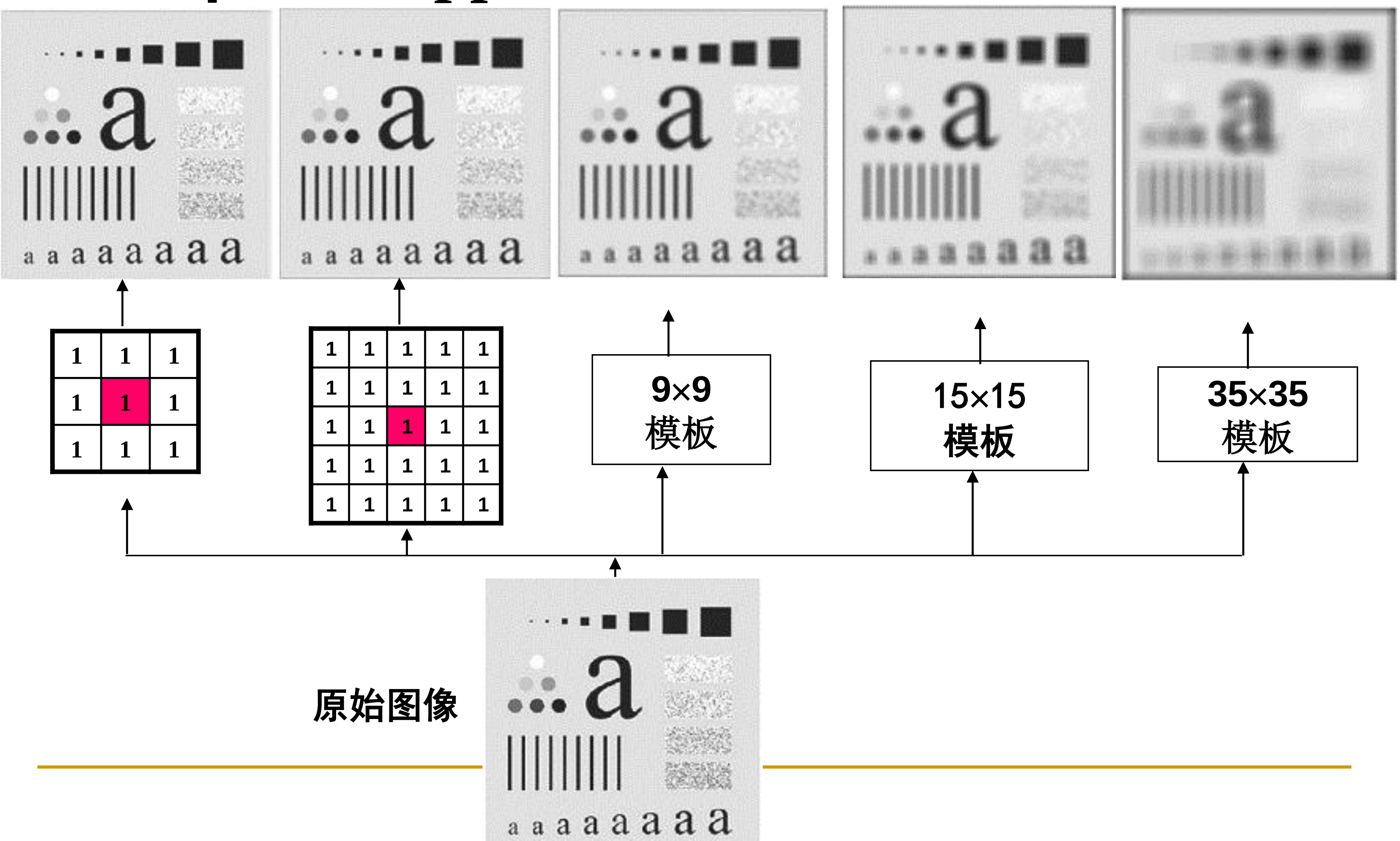
$\frac{1}{16}\times$

1	2	1
2	4	2
1	2	1

3.5.1 线性平滑滤波器

Smoothing Linear Filters

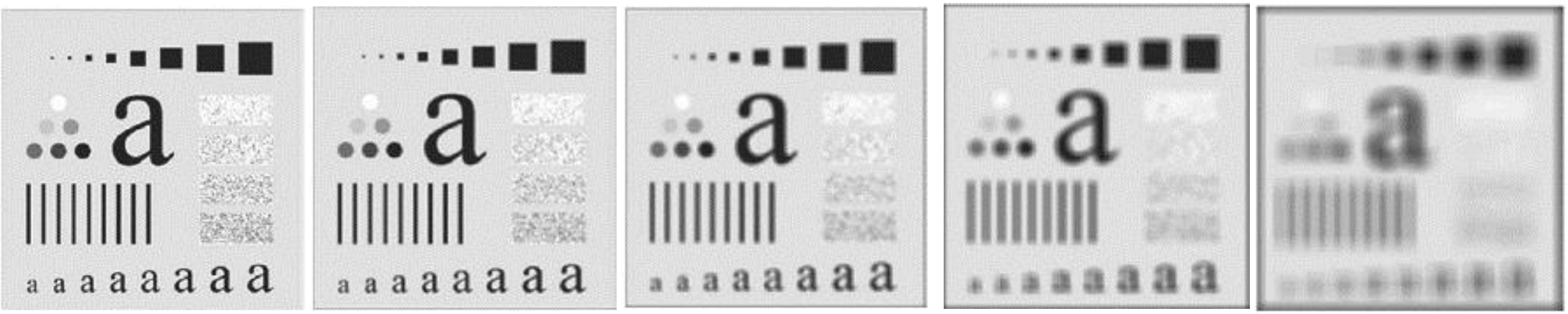
➤Example1: suppress noise but blur details(blending effe

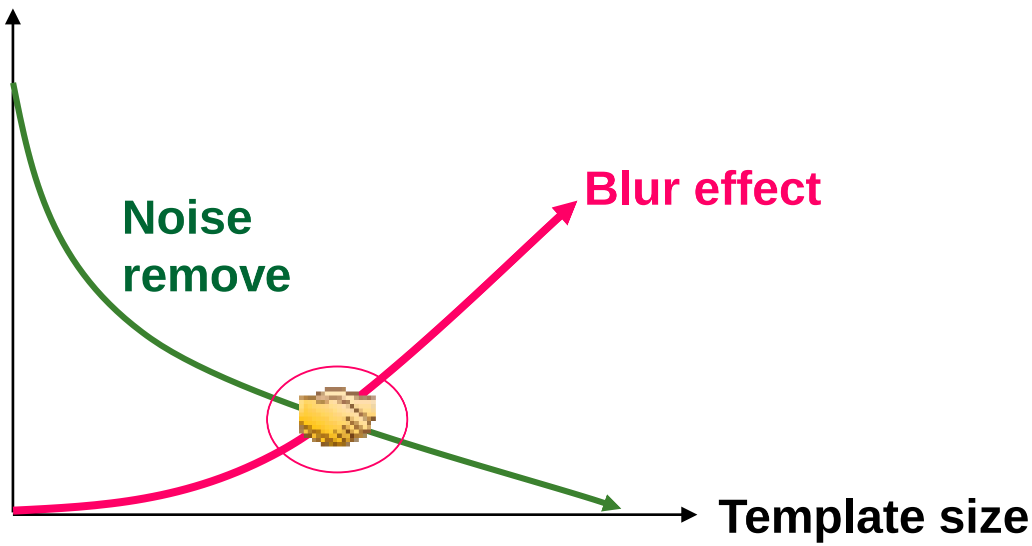


3.5.1 线性平滑滤波器

Smoothing Linear Filters

➢ **Example1: suppress noise but blur details (blending effect)**





3.5.1 线性平滑滤波器

Smoothing Linear Filters

- **Example2: suppress noise but blur details (blending effect)**

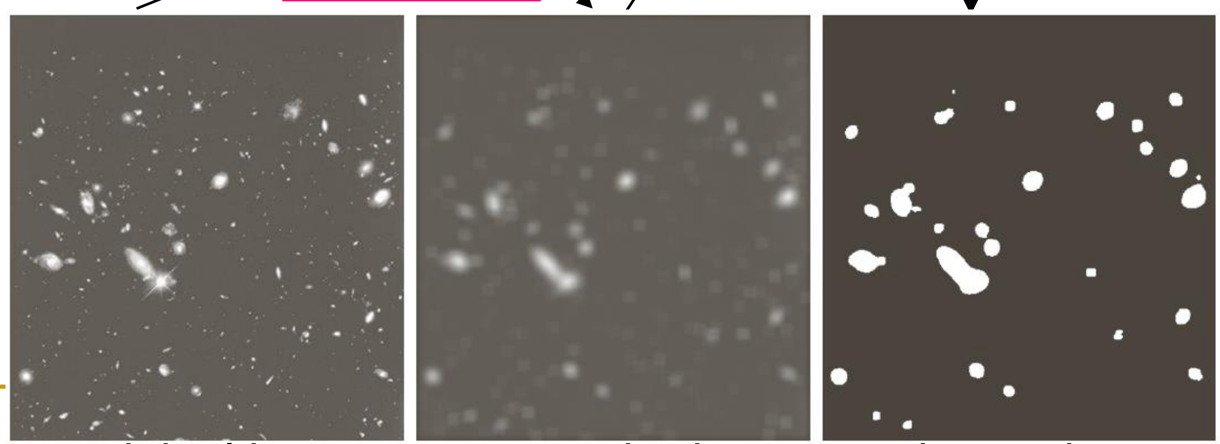
2) 输入原始图像 → **平滑滤波器 (模糊化,blurring 弱化小物体)** → **阈值化 (Thresholding)** → 输出增强图像 (强化大物体)

Basic idea: Big mask is used to eliminate small objects from an image. The size of the mask establishes the relative size of the objects that will be blended with the background.

15*15模板

thresholding阈值化(最大亮度的25%)

Hubble望远镜

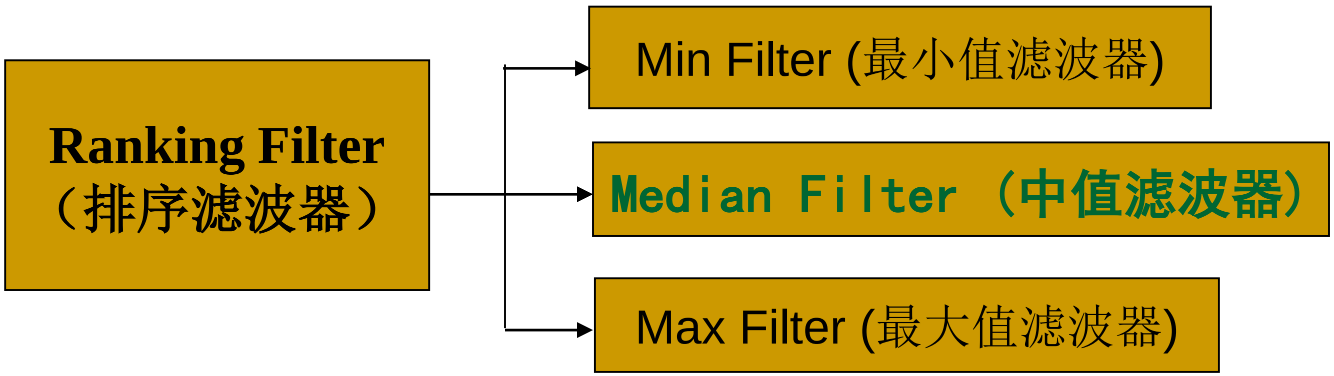


Original image　Smoothed　enhanced

3.5.2 非线性(排序)平滑滤波器

Smoothing Nonlinear Filters (Ranking Filter)

- **上述线性平滑滤波器虽然抑制了噪声，但同时也使图像的边缘也模糊了。**
- **Can we find a novel filter that can suppress noise while preserve details (edges)**
- **中值滤波器(Median Filter)：suppress noise+ preserve edges**



- Ranking Filter: The response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter

3.5.2 非线性(排序)平滑滤波器

Smoothing Nonlinear Filters (Ranking Filter)

✓中值滤波器(Median Filter)：抑制噪声的同时又能很好地保持图像的边缘

✓**中值滤波器**(median filter)：$R = \text{median}\{z_k \mid k = 1,2,\ldots, n \times n\}$

The median, ξ, of a set of values is such that half the values in the set are less than or equal to ξ, and half are greater than or equal to ξ.

Example：Given the intensities covered by the mask:

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

10	20	20
20	15	20
20	25	100

Please compute its median value

$\{z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8, z_9\}$

||

$\{10, 20, 20, 20, 15, 20, 20, 25, 100\}$

↓

Rank排序(从小到大)

↓

$\{10, 15, 20, 20, 20, 20, 20, 25, 100\}$

↓

求中值(第5个数的值)

↓

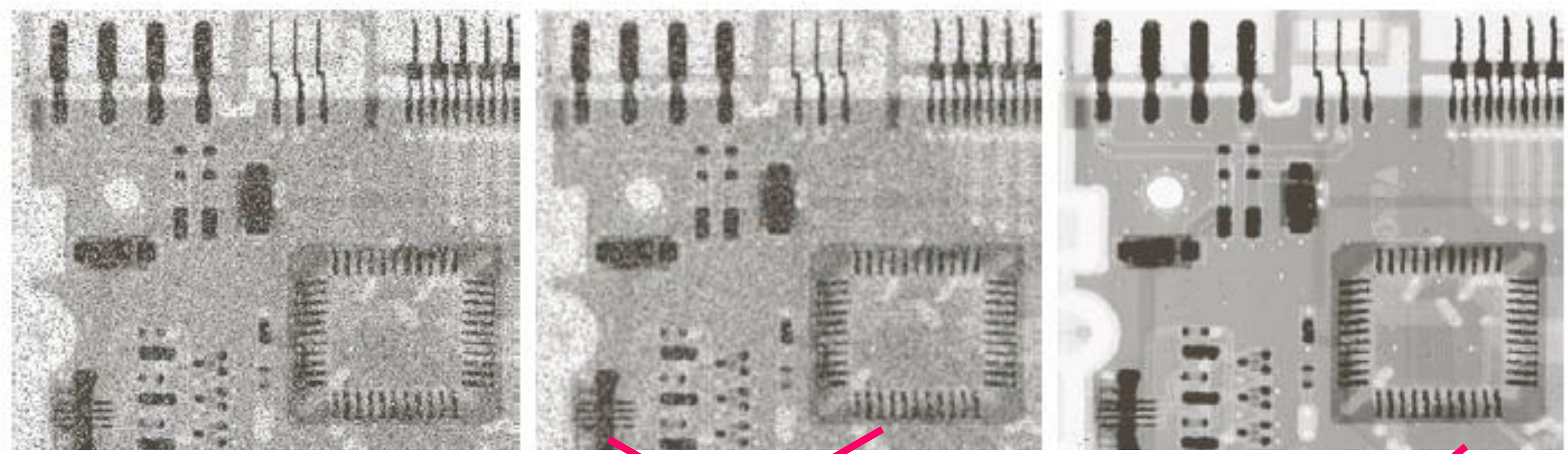
$R = median\{z_k\} = 20$

3.5.2 非线性(排序)平滑滤波器

Smoothing Nonlinear Filters (Ranking Filter)

✓**Example: Median filters for suppress salt-and-pepper noise**

椒盐噪声=impulse noise= white and black dots



Original input image　　3*3均值滤波器的结果　　中值滤波器的结果

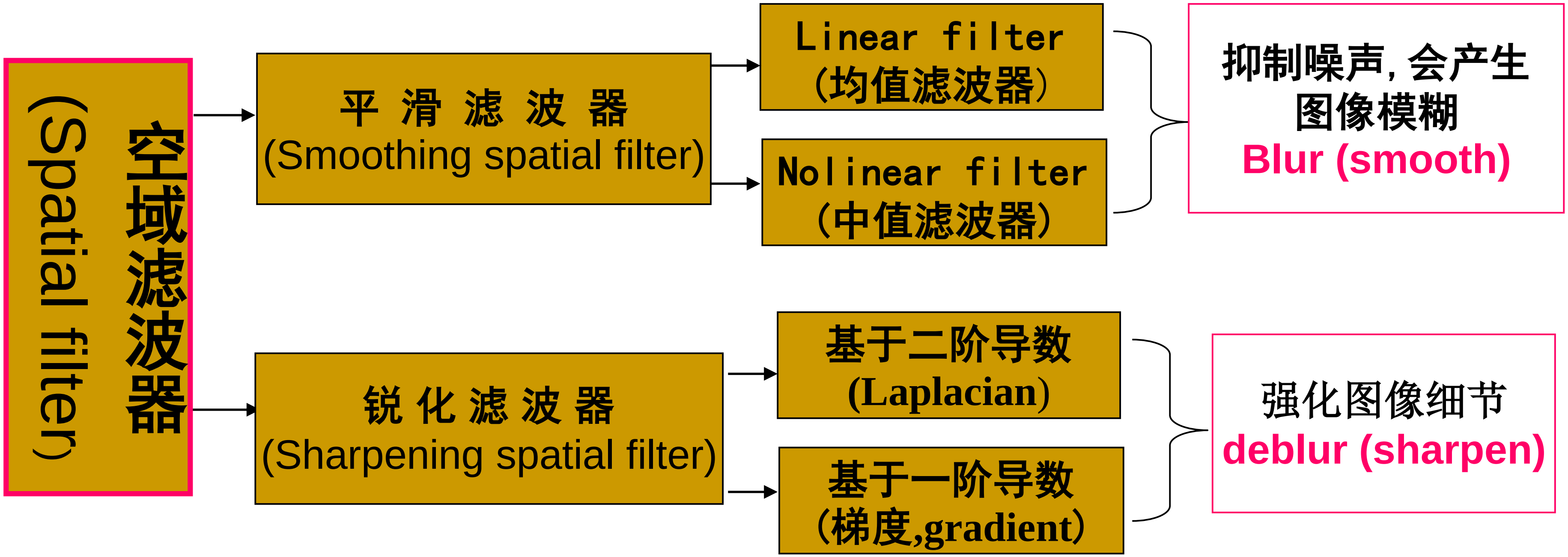
Incredible!

✓ **因此, 中值滤波器比均值滤波器更适合于加性椒盐噪声**

3.6 锐化滤波器

Sharpening Spatial Filters

✓**So far, we have learn:**



3.6.1 一阶和二阶导数

first- and second-order derivatives

- **Sharpening filter is based on the first & second derivative**

The first order derivative：$\left.\frac{\partial f(x)}{\partial x}\right|_{x=x_0} = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$ $, x \in \mathbb{R}$

$\Delta x = 1$

一阶导数近似: $\frac{\partial f(x)}{\partial x} = f(x+1) - f(x)$ $, x \in \mathbb{N}$

二阶导数近似: $$\frac{\partial^2 f}{\partial x^2} = f'(x) - f'(x-1)$$
$$= [f(x+1) - f(x)] - [f(x) - f(x-1)]$$
$$= f(x+1) + f(x-1) - 2f(x)$$

3.6.1 一阶和二阶导数

first- and second-order derivatives

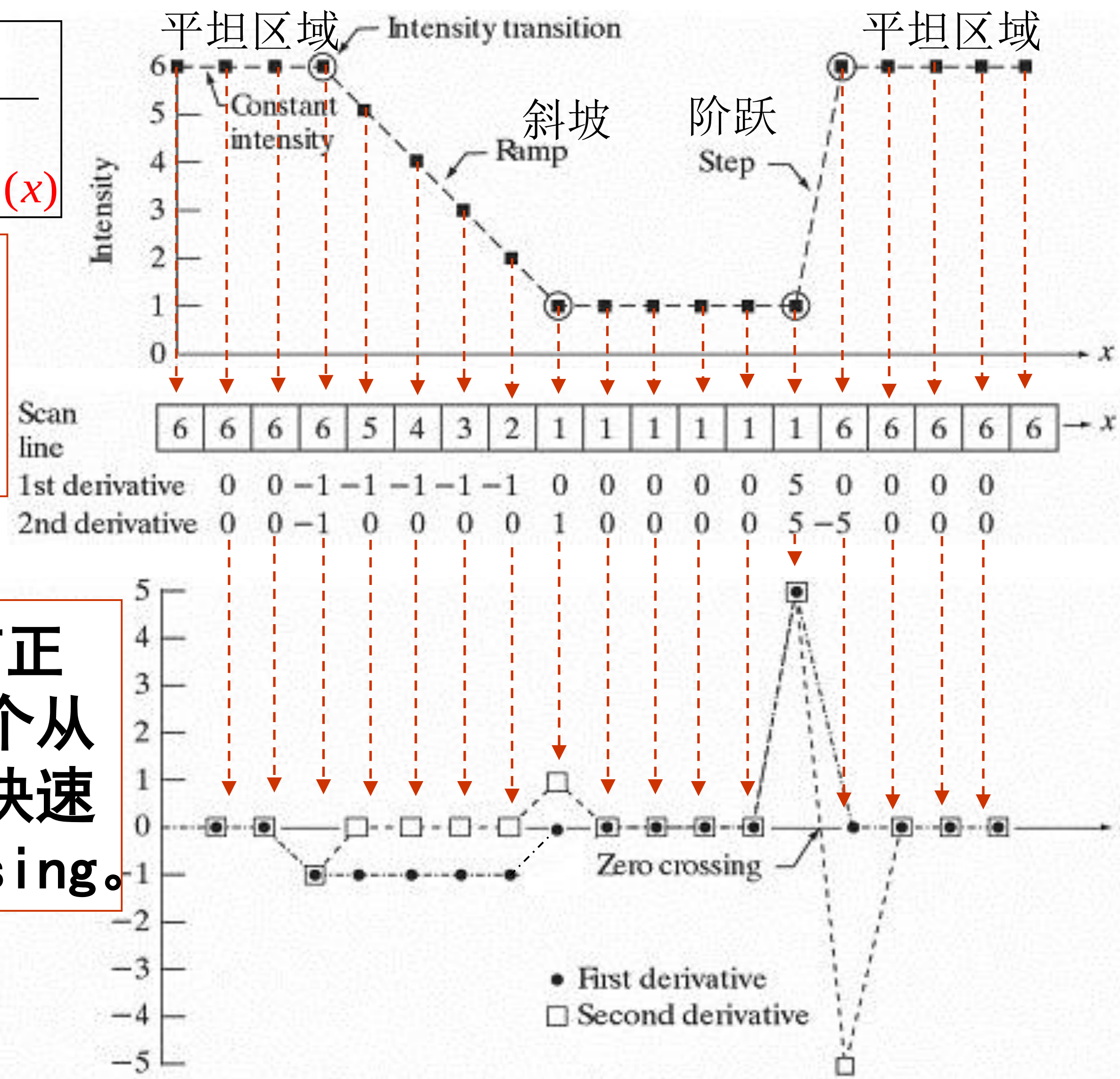
➢The features of the first & second order derivative

$$f'(x) = f(x+1) - f(x)$$

$$f''(x) = f'(x) - f'(x-1) = f(x+1) + f(x-1) - 2f(x)$$

斜坡区域: 一阶导数处处非零;二阶导数只在斜坡两端非零(上冲/下冲),其它部分均为零

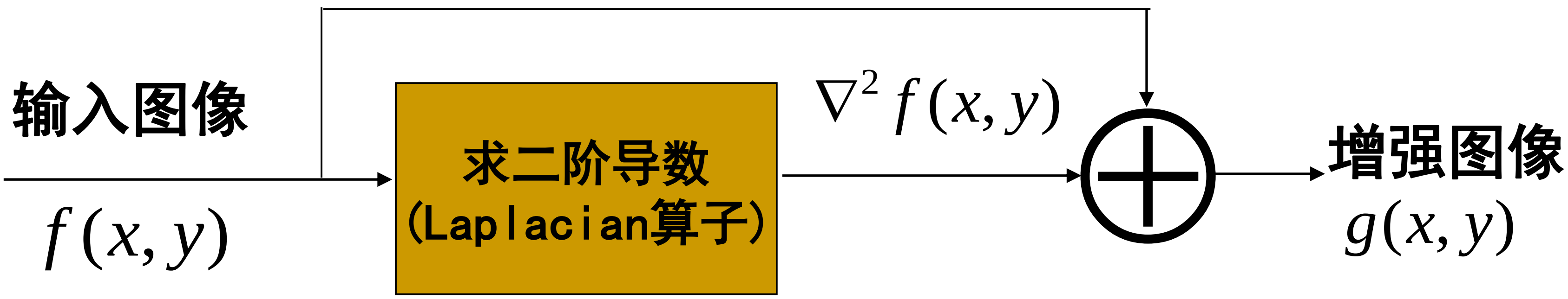
阶跃区域: 一阶导数有正突变；二阶导数有一个从正的极值到负极值的快速过渡,产生zero-crossing。



3.6.2 基于二阶导数的图像增强

Second Derivatives for Enhancement (the Laplacian)

➢basic idea



$$g(x,y) = f(x,y) + c[\nabla^2 f(x,y)]$$

How to compute $\nabla^2 f(x,y)$ **?**

3.6.2 基于二阶导数的图像增强

Second Derivatives for Enhancement (the Laplacian)

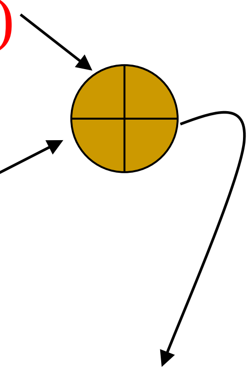
- 计算Laplacian算子 $\nabla^2 f(x,y)$
  - 函数 *f(x,y)* 的Laplacian算子定义为:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

  - 数字图像的二阶偏导为:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$



  - 故数字图像的Laplacian算子为:

$$\nabla^2 f = \left[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) \right] - 4f(x,y)$$

3.6.2 基于二阶导数的图像增强

Second Derivatives for Enhancement (the Laplacian)

➢The template of Laplacian operators

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y)$$

0	1	0
1	-4	1
0	1	0

对角拓展

1	1	1
1	-8	1
1	1	1

$\nabla^2 f =$ horizontal 二阶偏导 + vertical 二阶偏导

$\nabla^2 f =$ horizontal 二阶偏导 + vertical 二阶偏导 + 45°方向 二阶偏导 + 135°方向 二阶偏导

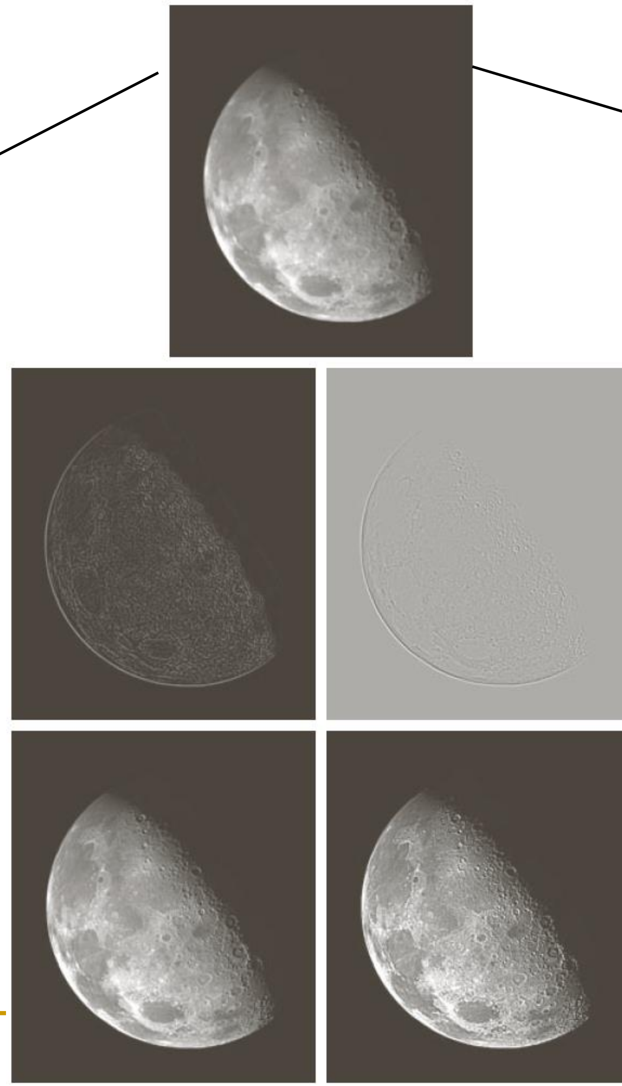
3.6.2 基于二阶导数的图像增强

Second Derivatives for Enhancement (the Laplacian)

➢Example: image sharpening (the north pole of the moon)

0	1	0
1	-4	1
0	1	0

$$g(x,y)=f(x,y)+\nabla^2 f(x,y)$$



1	1	1
1	-8	1
1	1	1

$$g(x,y)=f(x,y)+\nabla^2 f(x,y)$$

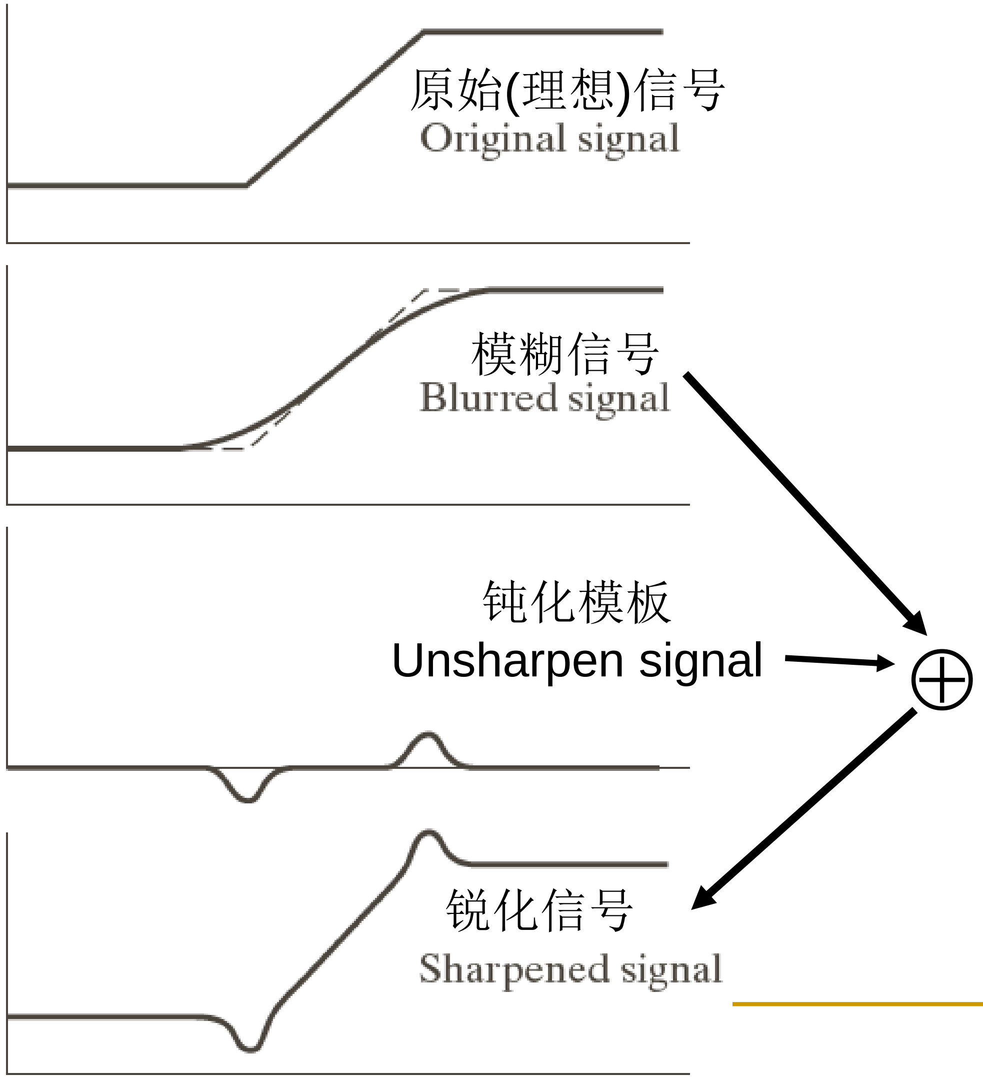
a
b c
d e

FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

3.6.3 非锐化掩蔽和高提升滤波

Unsharp Masking and Highboost Filter

➢Intuitive idea of image sharpening (enhancement)



3.6.4 基于一阶导数的图像增强

First Derivatives for Enhancement (Gradient, Sobel)

➢用梯度来锐化图像

输入图像 $f(x,y)$ → 与梯度算子卷积 (Sobel算子,Roberts算子) → 增强图像 $g(x,y)=\nabla f(x,y)$

函数的梯度是一个向量：

$$\nabla \mathbf{f} = \left[\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \right]^T$$

常用梯度的幅度代替梯度本身：

$$\nabla f = mag(\nabla \mathbf{f}) = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

常用绝对值代替平方：

$$\nabla f = mag(\nabla \mathbf{f}) \approx \left| \frac{\partial f}{\partial x} \right| + \left| \frac{\partial f}{\partial y} \right|$$

3.6.4 基于一阶导数的图像增强

First Derivatives for Enhancement (Gradient, Sobel)

- $\nabla f \approx \left|\frac{\partial f}{\partial x}\right| + \left|\frac{\partial f}{\partial y}\right| = |G_x| + |G_y|$ convolution mask

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

① Roberts operator(1965年)

$$\nabla f \approx |G_x| + |G_y| \approx |z_9 - z_5| + |z_8 - z_6|$$

-1	0
0	1

0	-1
1	0

The Roberts operator is one of the oldest operators [Roberts 65]

The primary disadvantage of Roberts operator is its high sensitivity to noise, because very few pixels are used to approximate the gradient

② Sobel operator

$$\nabla f \approx |G_x| + |G_y|$$
$$= |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)|$$
$$+ |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

The idea behind using a weight value of 2 is to achieve some smoothing by giving more importance to the center point

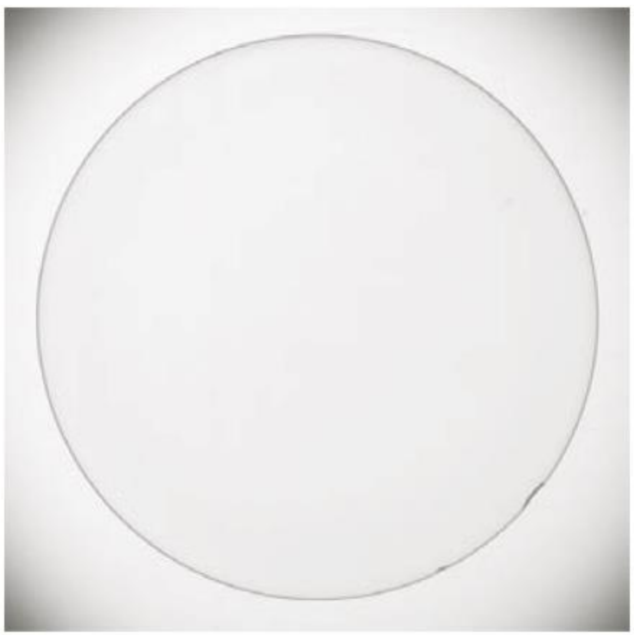
3.6.4 基于一阶导数的图像增强

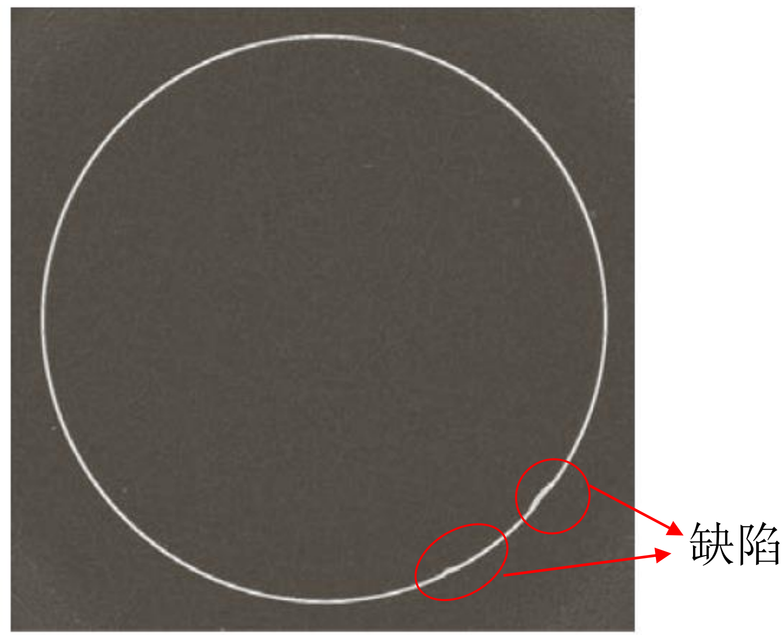
First Derivatives for Enhancement (Gradient, Sobel)

- **Sobel 梯度算子应用实例（contact lens,隐形眼镜）**

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1





3.6 锐化滤波器

Sharpening Spatial Filters

- **微分算子小结**
  - **一阶微分算子：①突出小缺陷；②去除慢变化背景**
  - **二阶微分算子：增强灰度突变处的对比度**

Summary

- Background
- Some basic intensity transformation
- Histogram-based image enhancement
- Fundamentals of spatial filtering
- Smoothing spatial filters
- Sharpen spatial filters

作业：见发布的word文档