频率域滤波

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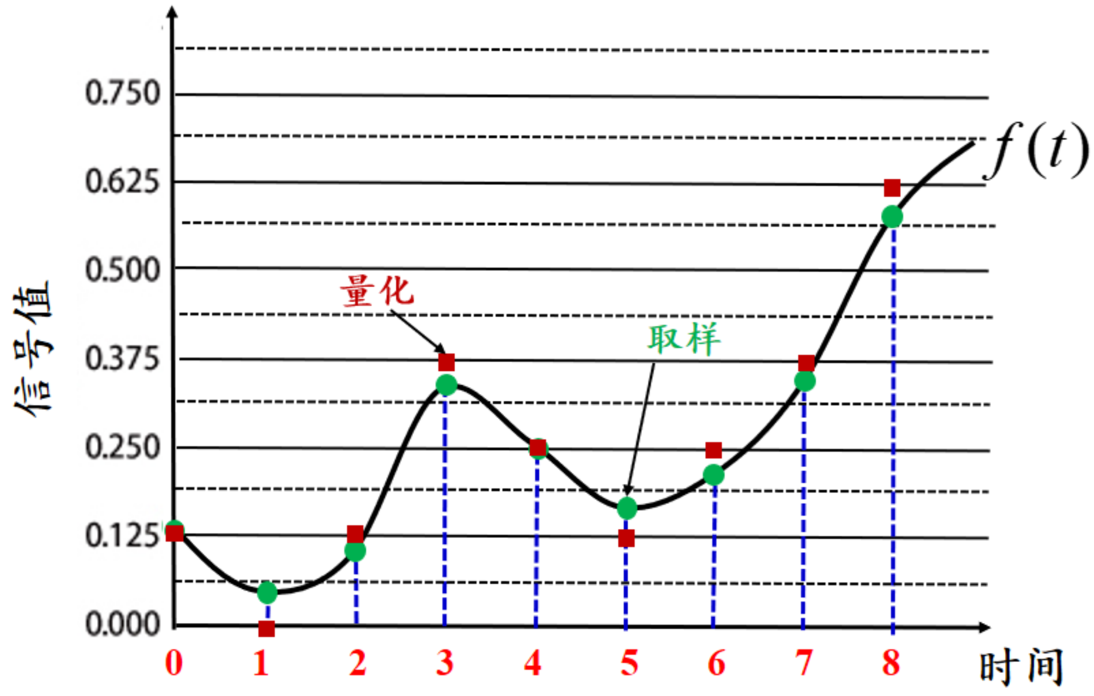
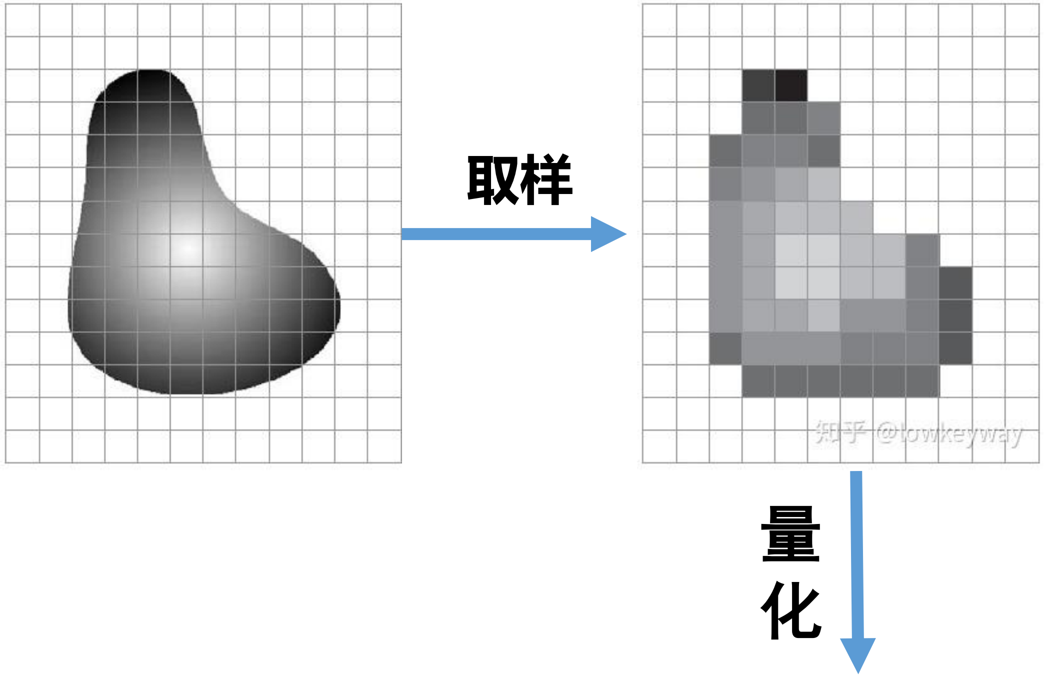
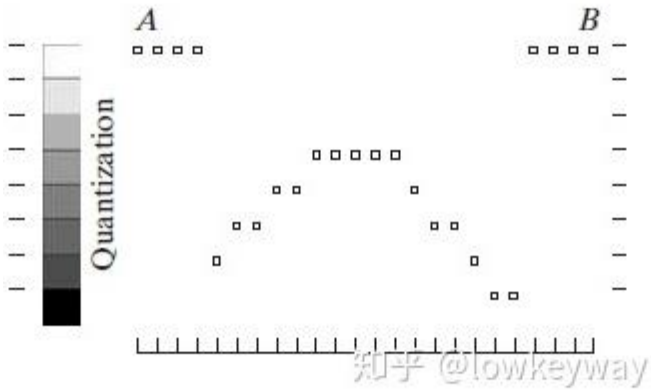
§4.3 取样和取样函数的傅立叶变换

模拟信号与数字信号





模拟图像与数字图像



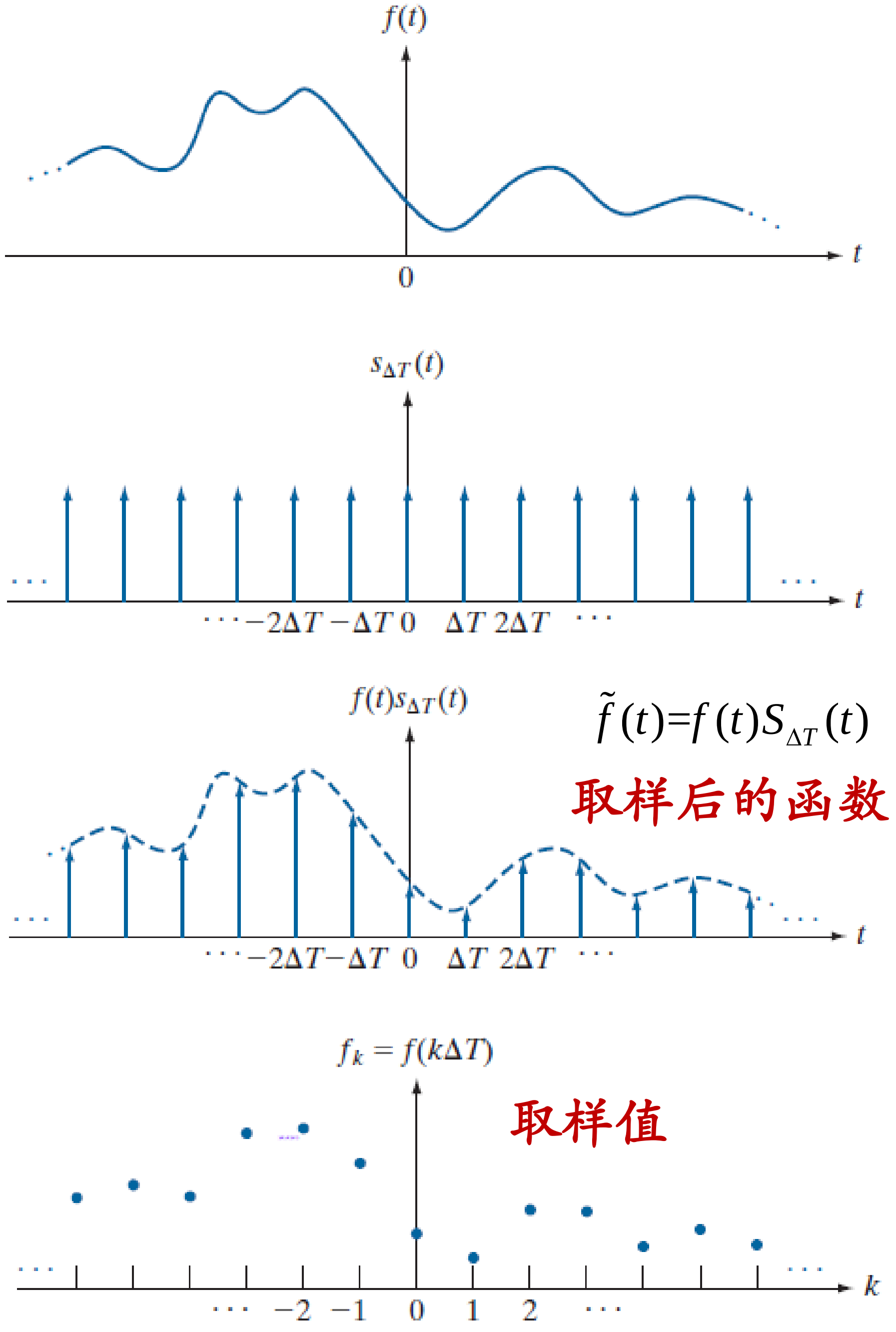
§4.3.1 取样

取样函数用以均匀间隔(ΔT)的冲击串代替，因此模拟取样的结果是用取样函数同连续函数相乘

$$\tilde{f}(t) = f(t)S_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T)$$

序列中的任意取样值 f_k 为

$$\boldsymbol{f_k = \int_{-\infty}^{\infty} f(t)\delta(t - k\Delta T)dt = f(k\Delta T)}$$



§4.3.2 取样函数的傅立叶变换

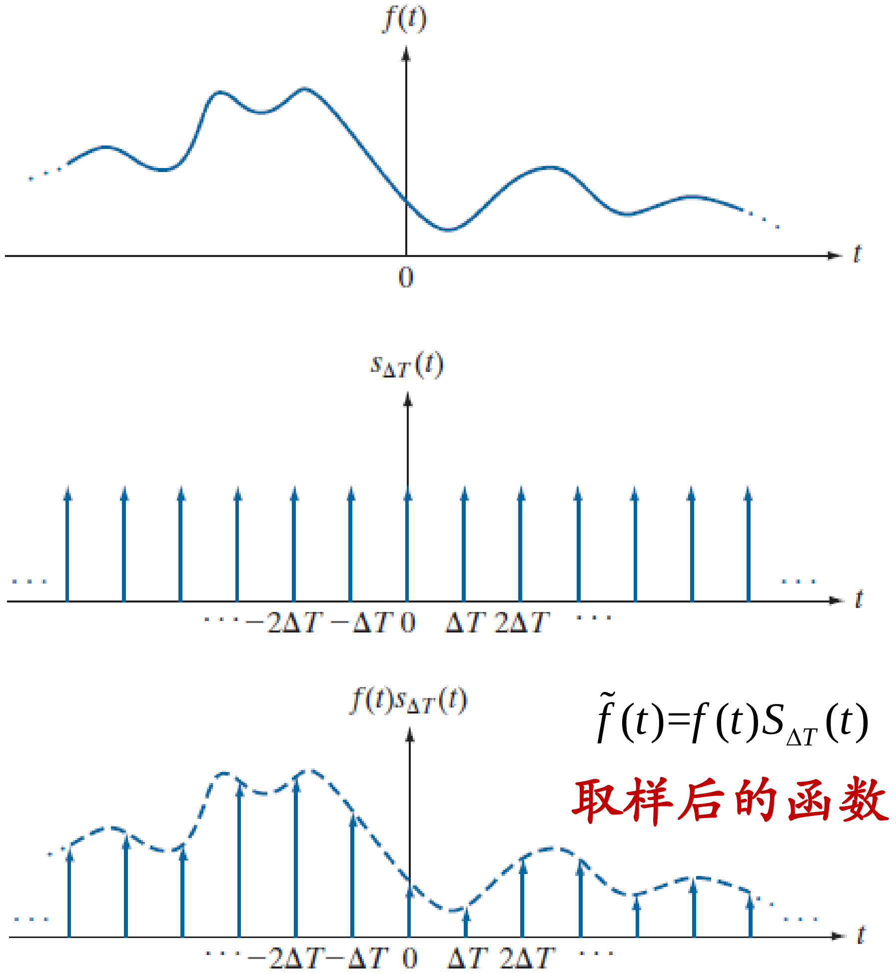
$$\tilde{f}(t) = f(t)s_{\Delta T}(t)$$

$$\tilde{F}(\mu) = \mathfrak{J}\{\tilde{f}(t)\} = \mathfrak{J}\{f(t)s_{\Delta T}(t)\} = F(\mu)\bigstar S(\mu)$$

$$\tilde{F}(\mu) = F(\mu)\bigstar S(\mu)$$

$$= \int_{-\infty}^{\infty} F(\tau)S(\mu-\tau)d\tau$$

$$= \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} F(\mu - \frac{n}{\Delta T})$$

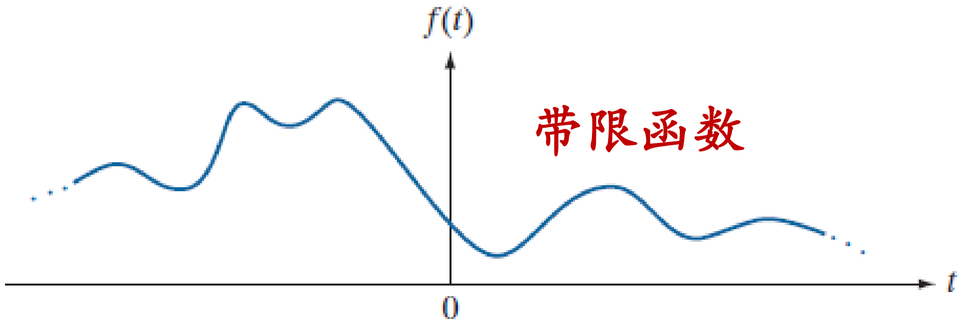


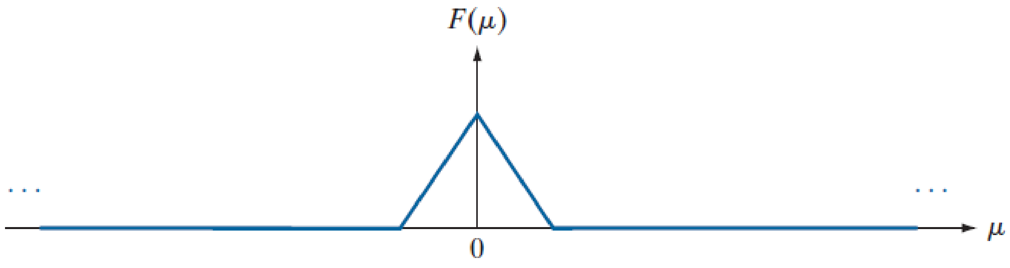
冲激串的傅里叶变换

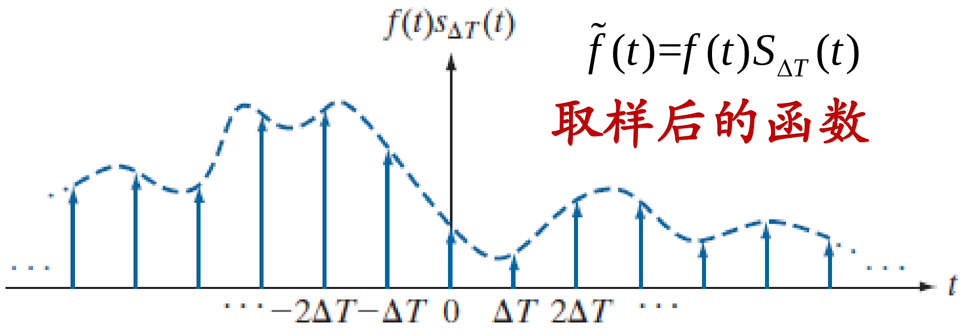
$$s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T) \Rightarrow S(\mu) = \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} \delta(\mu - \frac{n}{\Delta T})$$

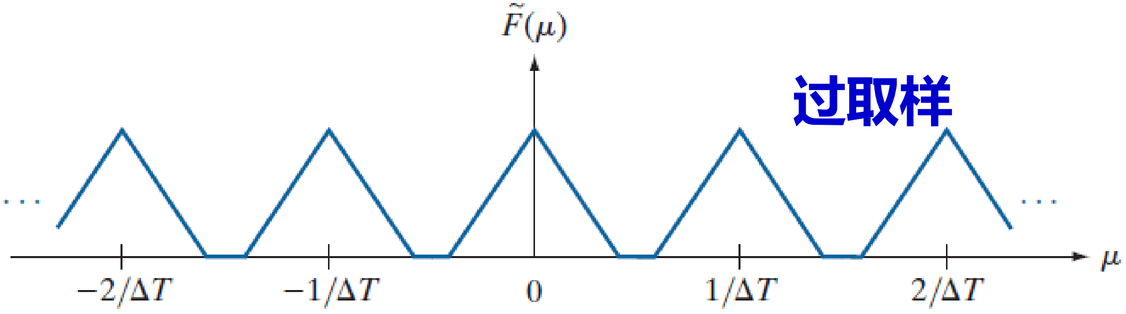
§4.3.2 取样函数的傅立叶变换

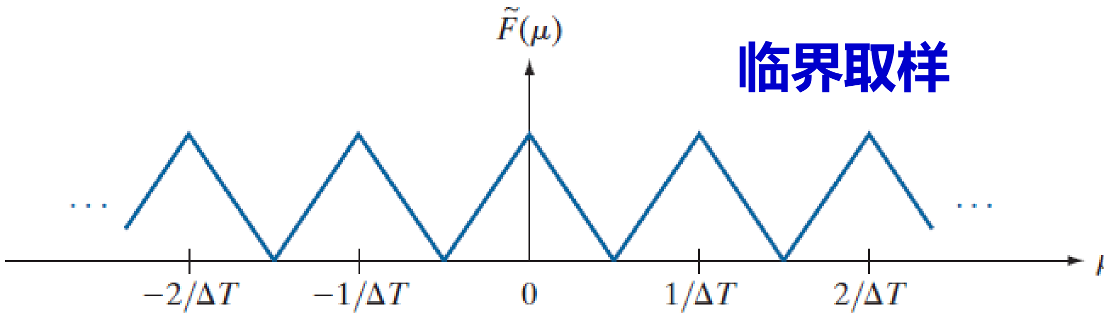
$$\tilde{F}(\mu) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} F\left(\mu - \frac{n}{\Delta T}\right)$$

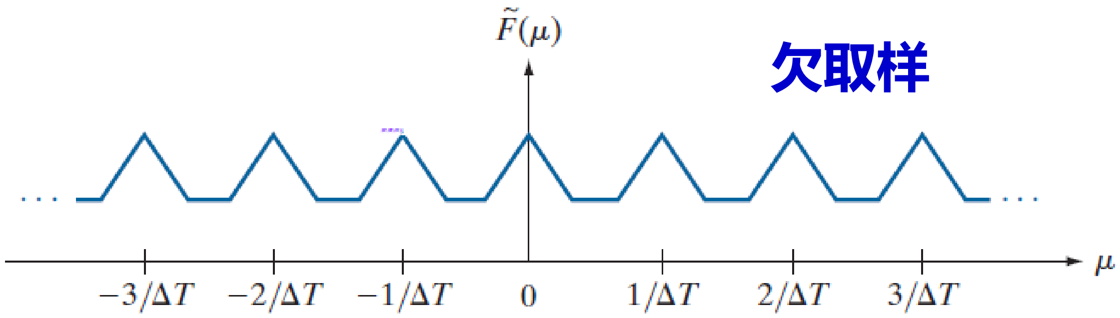






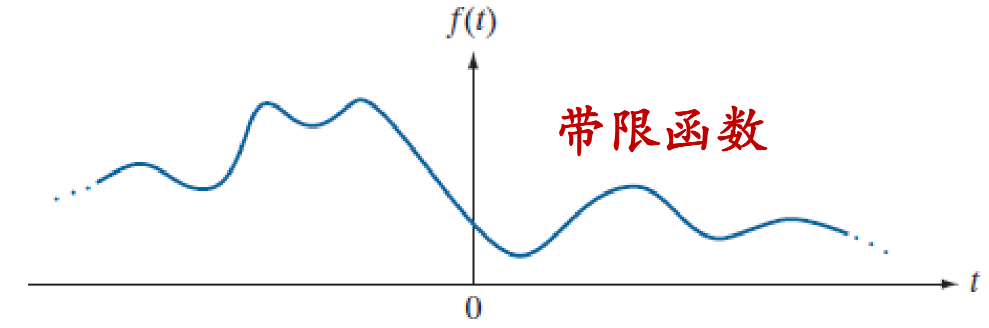


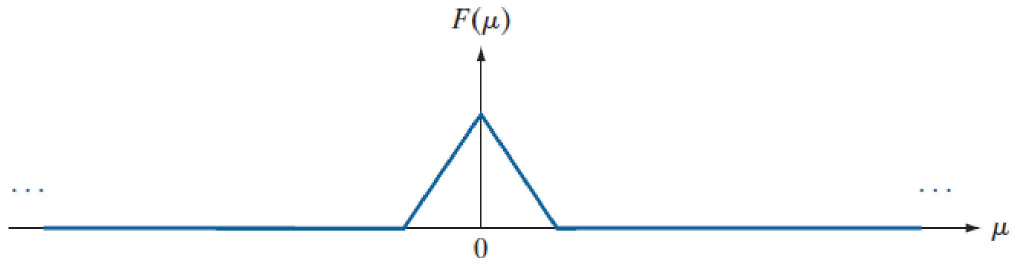


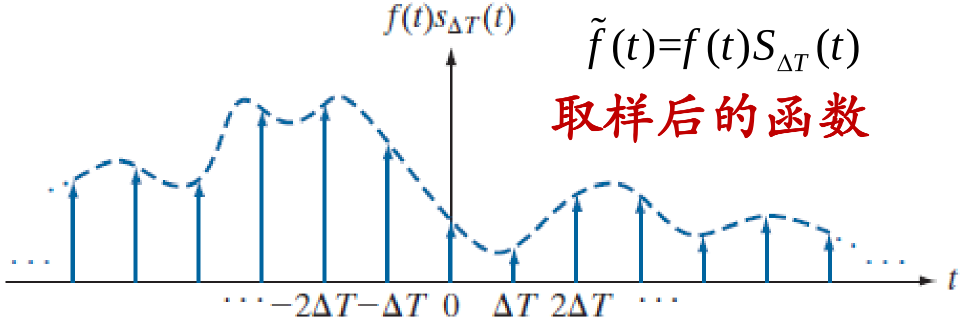


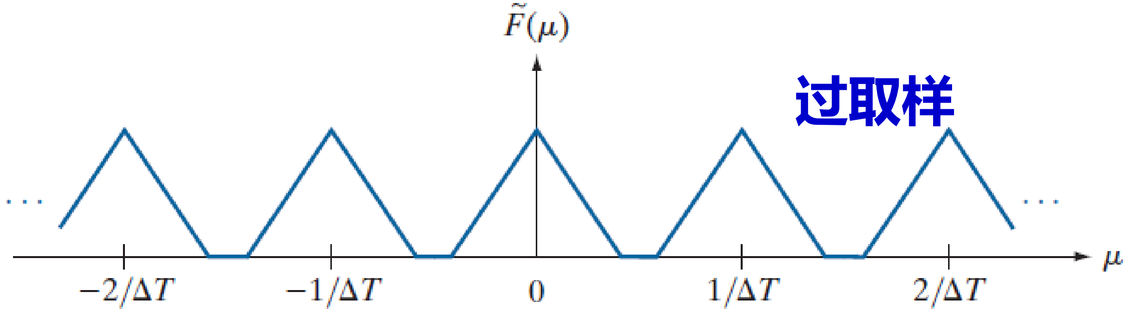
§4.3.2 取样函数的傅立叶变换

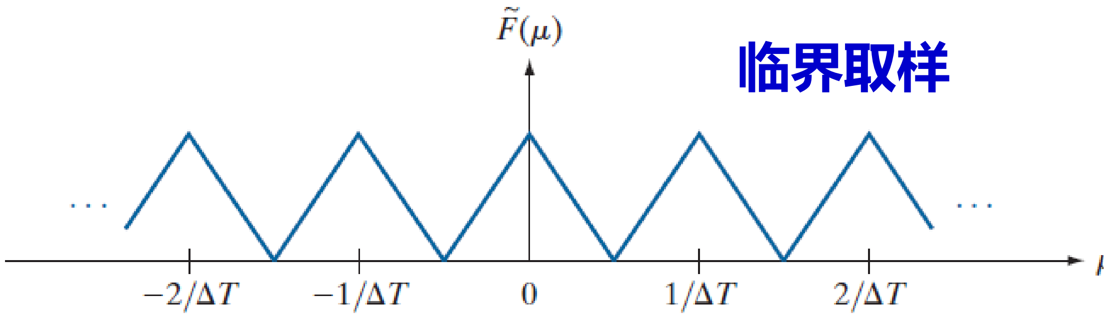
1/ΔT是取样函数的取样率。取样率必须足够高，才能使得不同周期之间有有效的间隔，

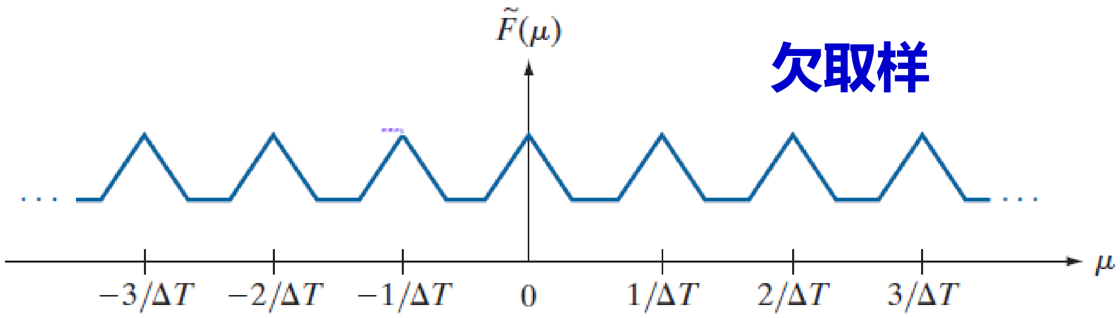






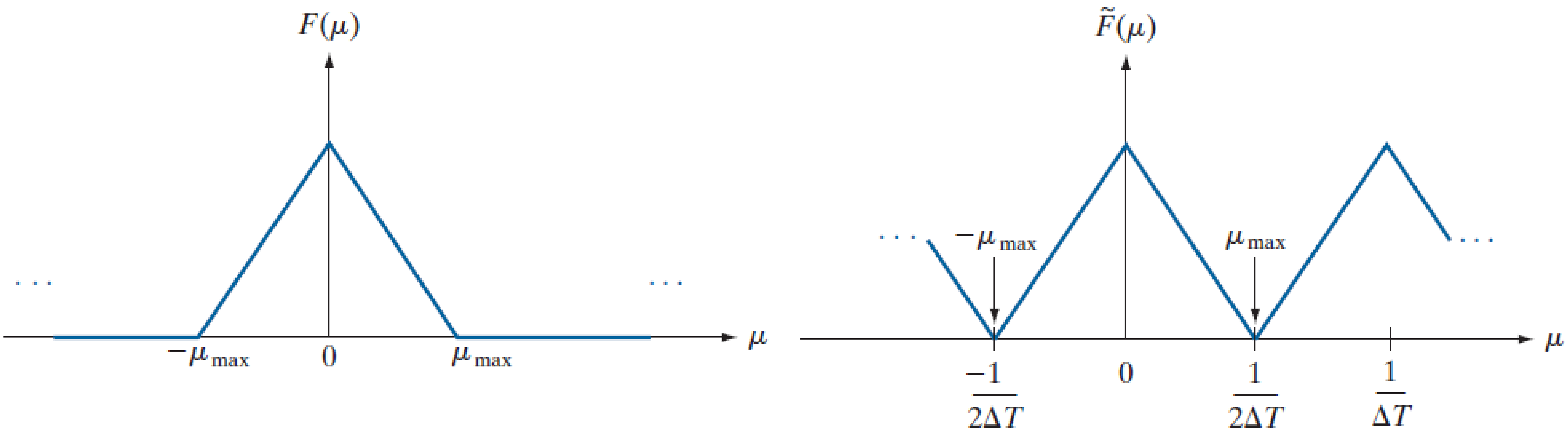






§4.3.3 取样定理

如何给出适当条件，使得一个连续函数完全可由其取样集合来恢复?



取样定理
奈奎斯特准则

$$\frac{1}{\Delta T} > 2\mu_{max}$$

取样率非常关键

§4.3.3 取样定理

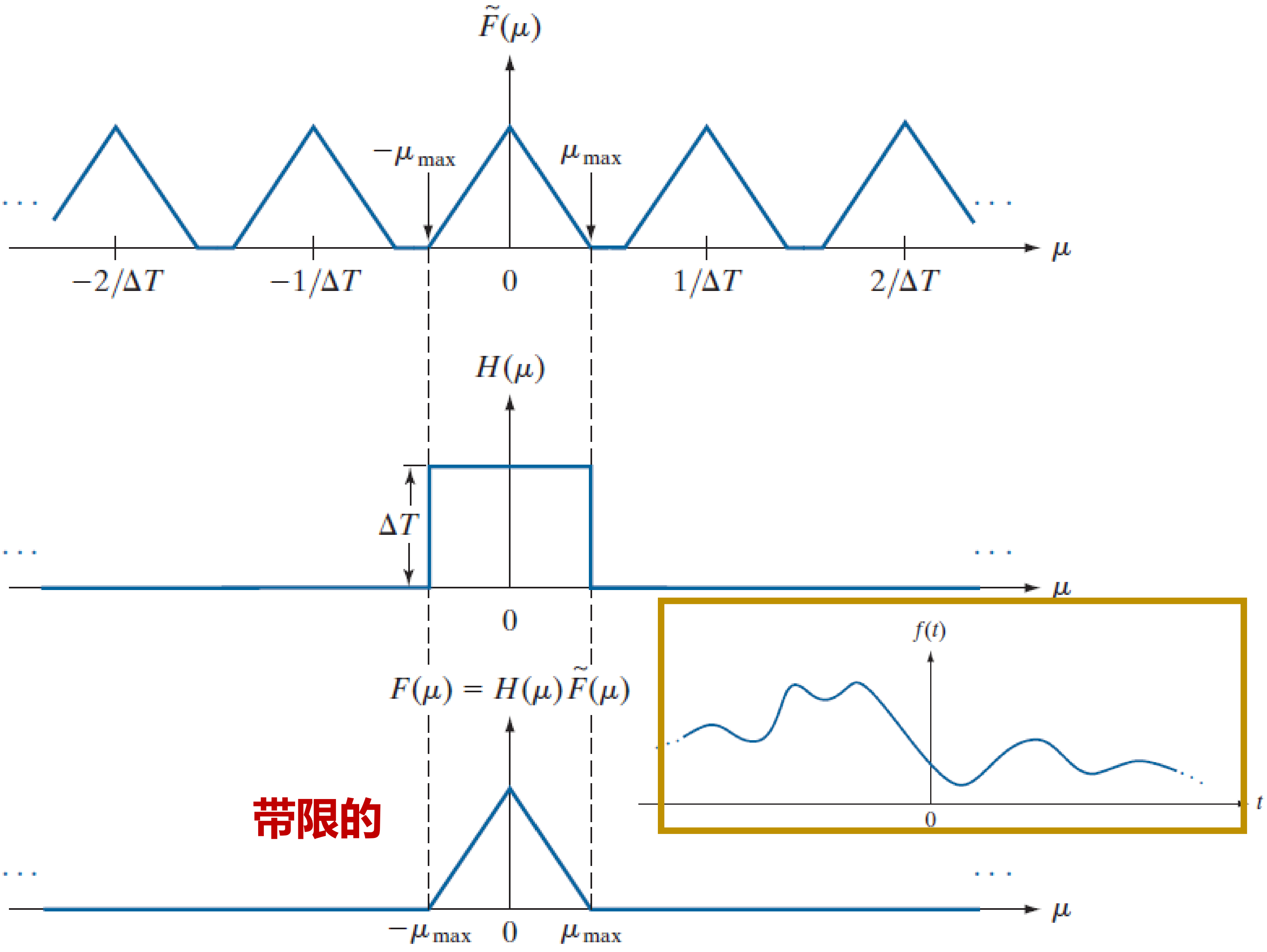
- **恢复/复原过程**

$$F(\mu) = H(\mu)\tilde{F}(\mu)$$

$$f(t) = \int_{-\infty}^{\infty} F(\mu)e^{j2\pi\mu t}d\mu$$

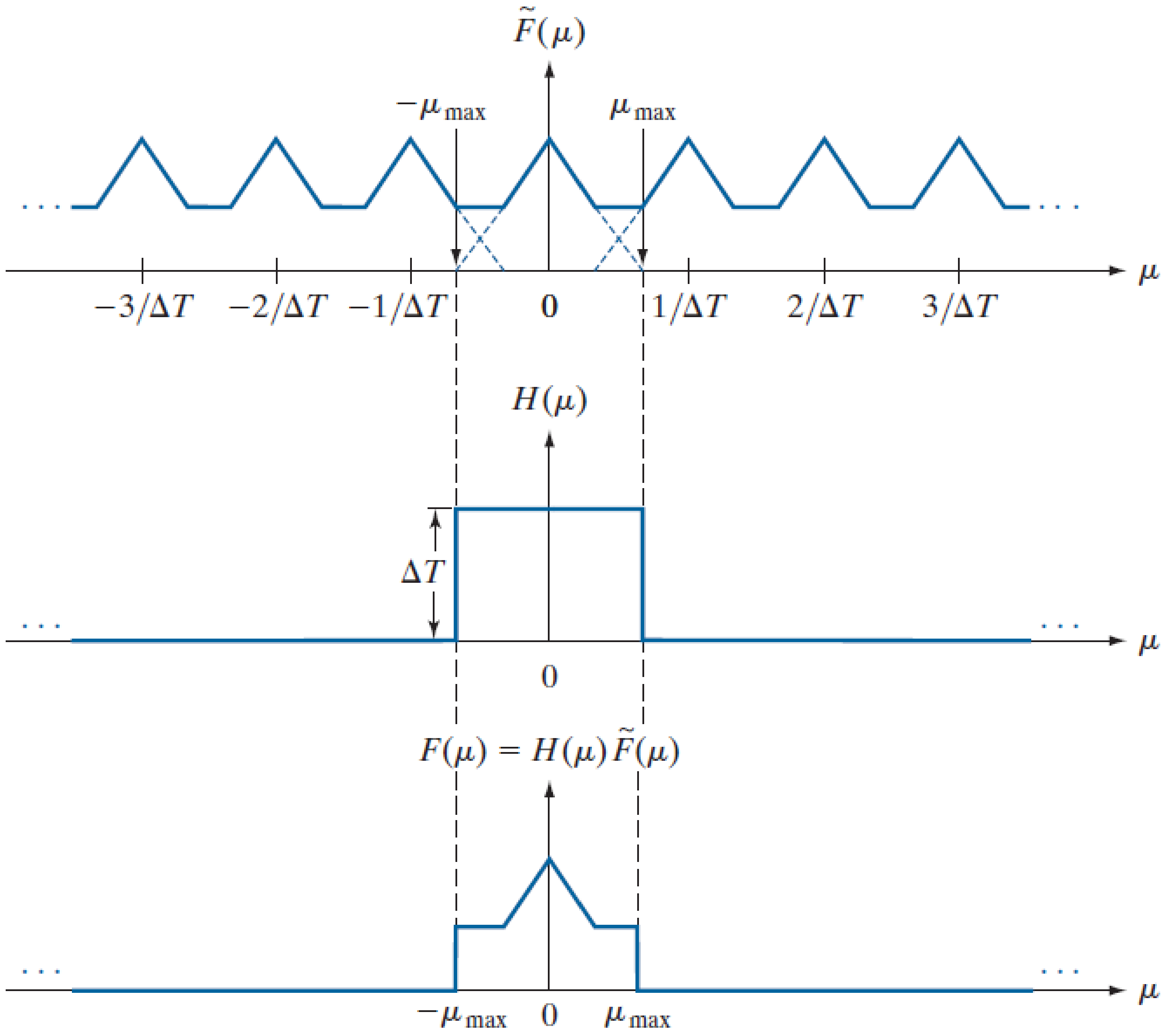
- $f(t)$必须是带限的，即必须从$-\infty$扩展到∞

- 限制一个函数的持续时间会妨碍该函数的完美复原



§4.3.4 混淆

频率混淆是由函数欠采样导致



混淆在所难免

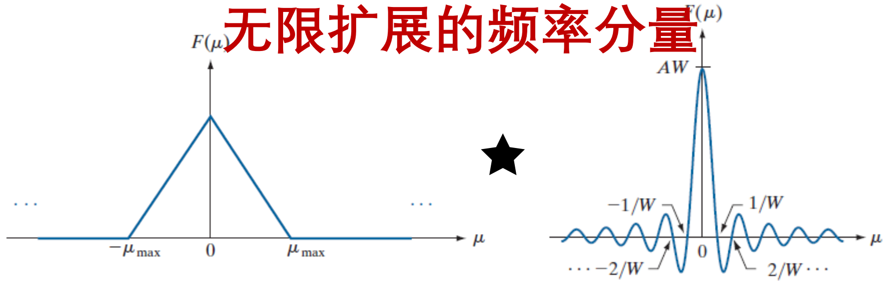
实际中限制函数的时间，通过将持续时间限制在区间[0,T]内，$f(t)h(t)$

$$\boxed{h(t) = \begin{cases} 1, 0 \le t \le T \\ 0, others \end{cases}}$$

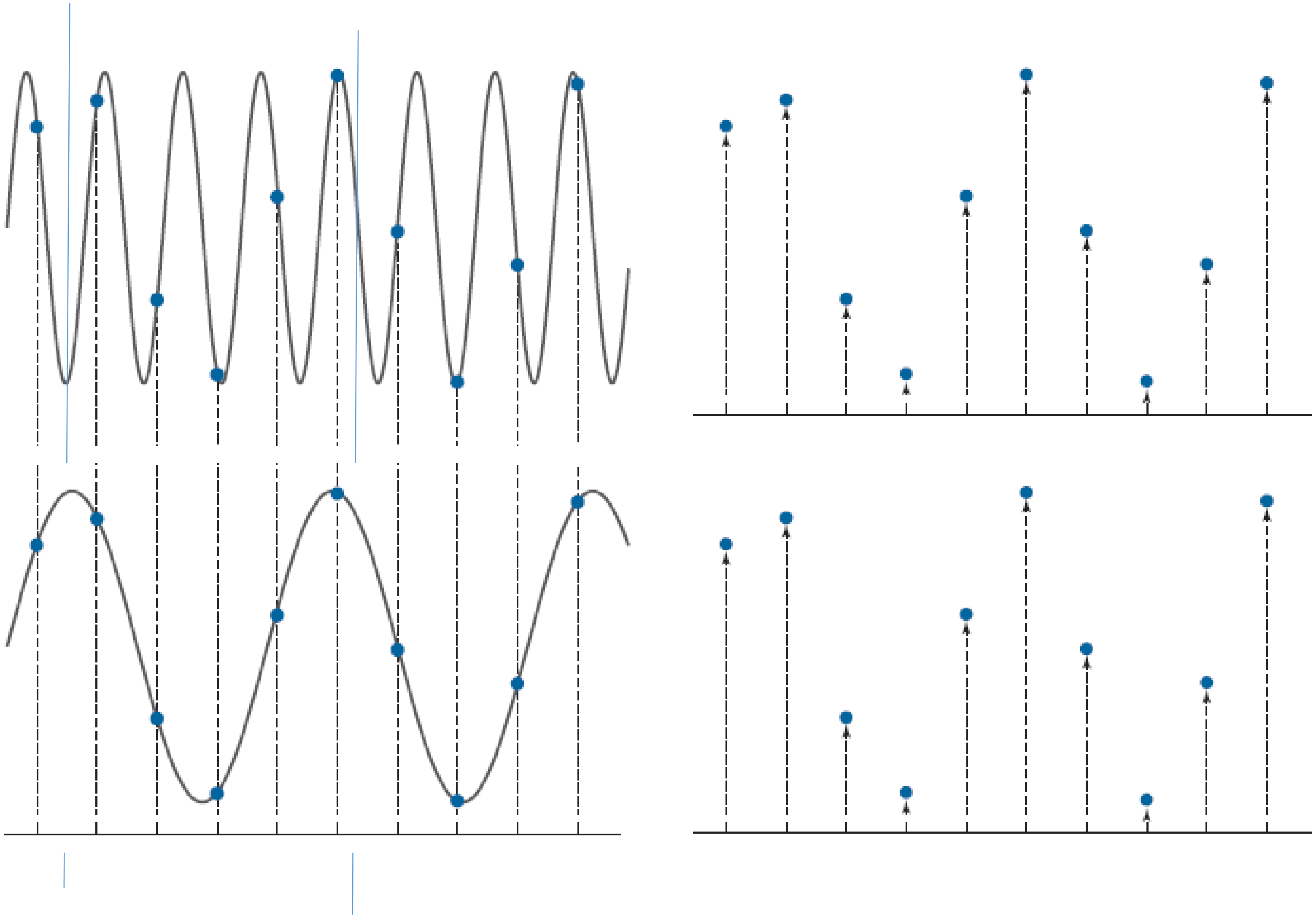
$f(t)h(t)$的变换等于两个函数变换的卷积

$$f(t)h(t) \Rightarrow F(\mu)\bigstar H(\mu)$$

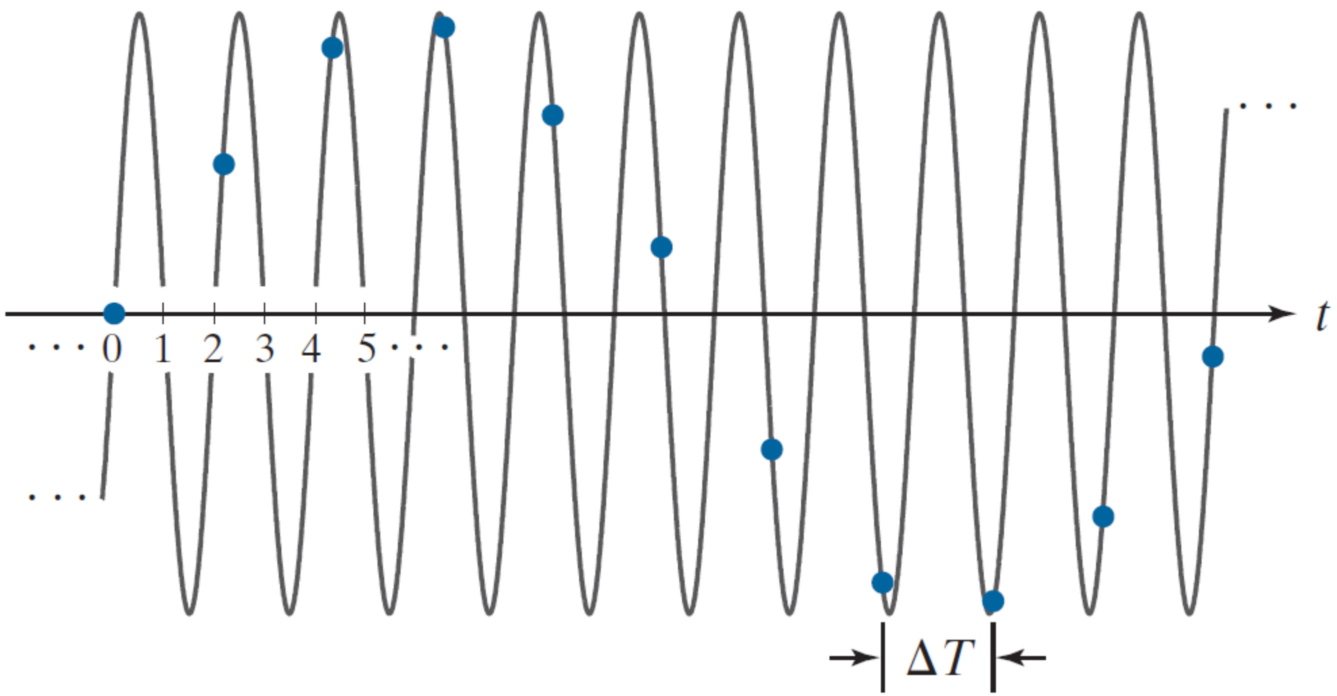
无限扩展的频率分量



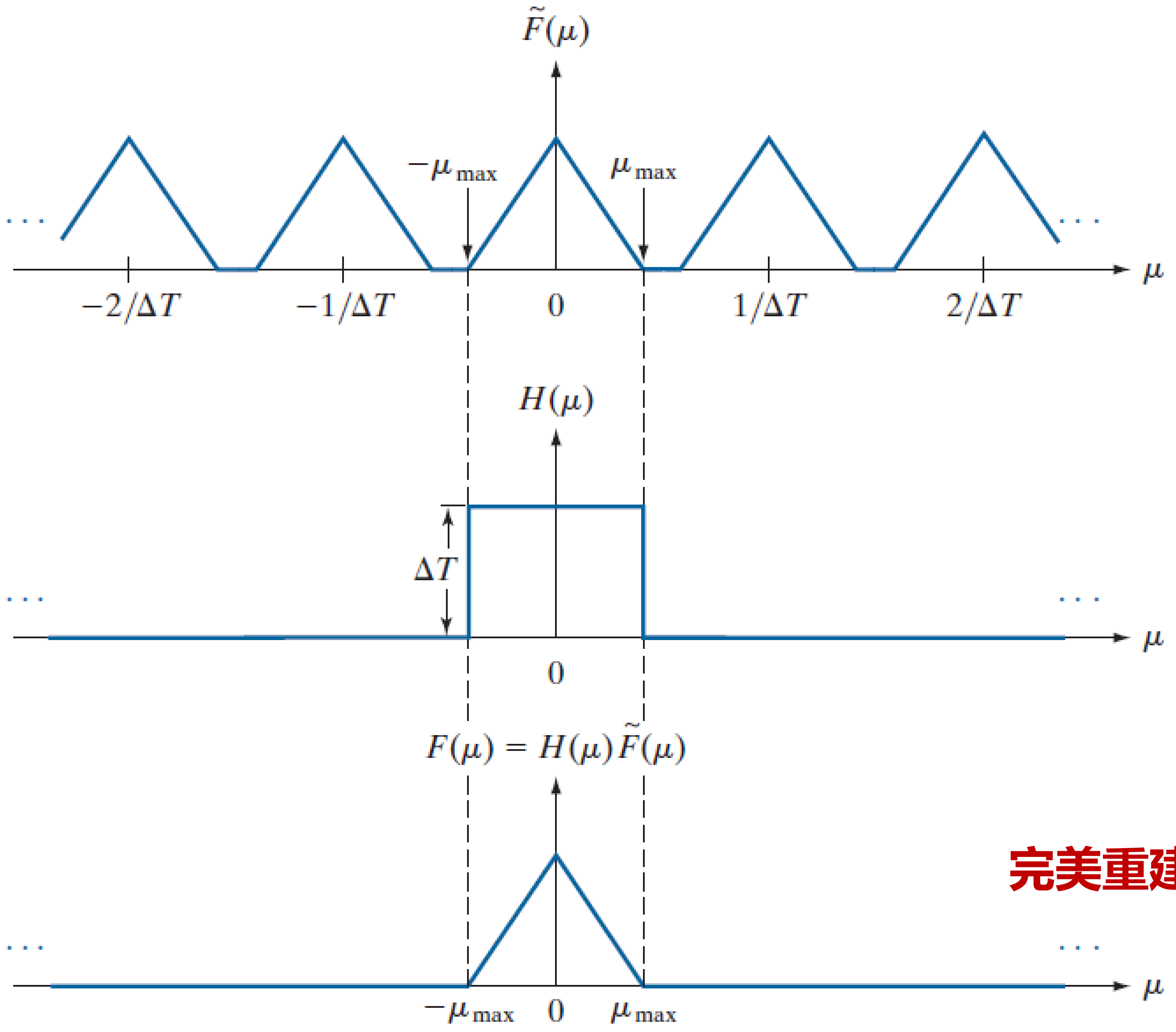
§4.3.4 混淆



§4.3.4 混淆



§4.3.5 由取样后的数据重建（复原）函数

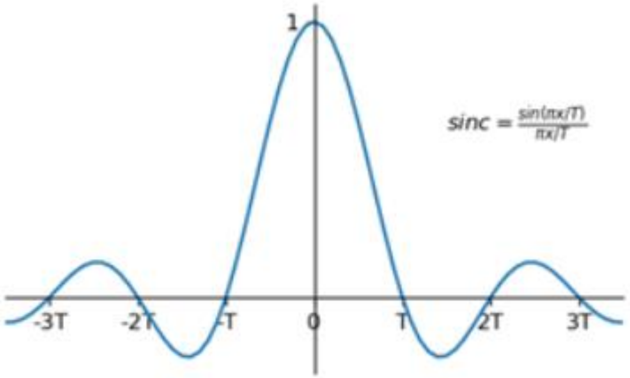


$$F(\mu) = H(\mu)\tilde{F}(\mu)$$

$$\begin{aligned} f(t) &= \mathfrak{F}^{-1}\{F(\mu)\} \\ &= \mathfrak{F}^{-1}\{H(\mu)\tilde{F}(\mu)\} \\ &= h(t) \star \tilde{f}(t) \\ &= \sum_{n=-\infty}^{\infty} f(n\Delta T)\,sinc\left(\frac{t-n\Delta T}{\Delta T}\right) \end{aligned}$$

完美重建函数是用取样值加权的sinc函数的无限和

§4.3.5 由取样后的数据重建（复原）函数



$$f(t) = \sum_{n=-\infty}^{\infty} f(n\Delta T) sinc\left(\frac{t - n\Delta T}{\Delta T}\right)$$