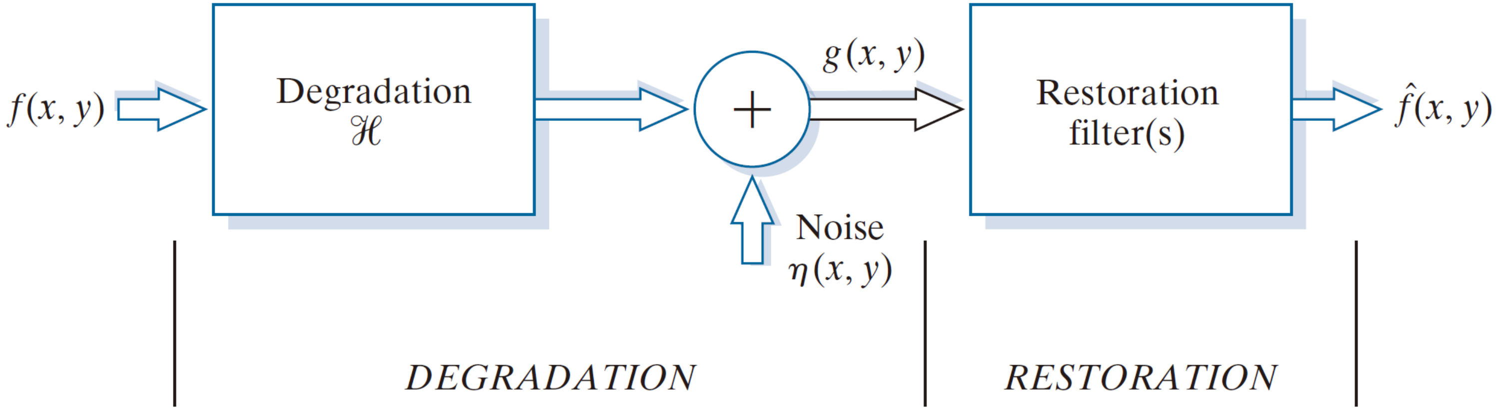
主要内容

- 图像退化/复原过程的模型
- 噪声模型
- 空间域滤波方法
- 频率域滤波方法
- 退化函数的估计
- 逆滤波
- 维纳滤波

5.3 只存在噪声的复原 —— 空间滤波

FIGURE 5.1
A model of the image degradation/restoration process.



$$g(x,y) = f(x,y) \star h(x,y) + \eta(x,y)$$

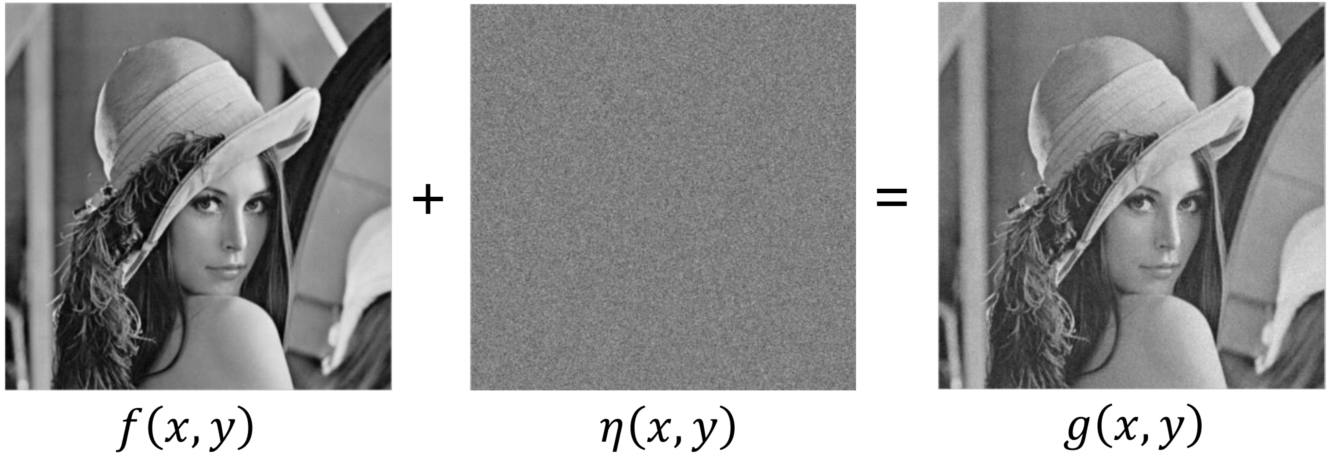
$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

$$g(x,y) = f(x,y) + \eta(x,y)$$

$$G(u,v) = F(u,v) + N(u,v)$$

加性噪声

- 去噪 denoising

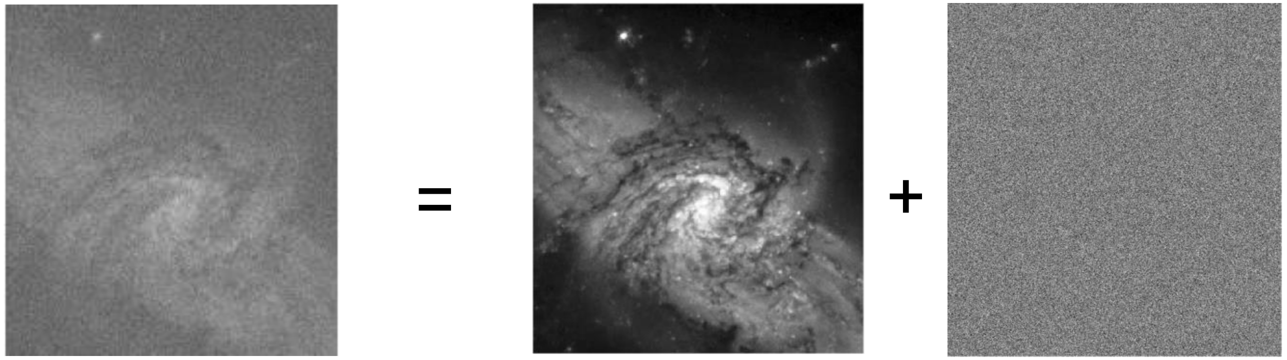


$f(x,y)$ $\eta(x,y)$ $g(x,y)$

图像降噪 —— 噪声图像相加（平均）

$$g(x,y) = f(x,y) + \eta(x,y)$$

退化的图像　　理想图像　　噪声

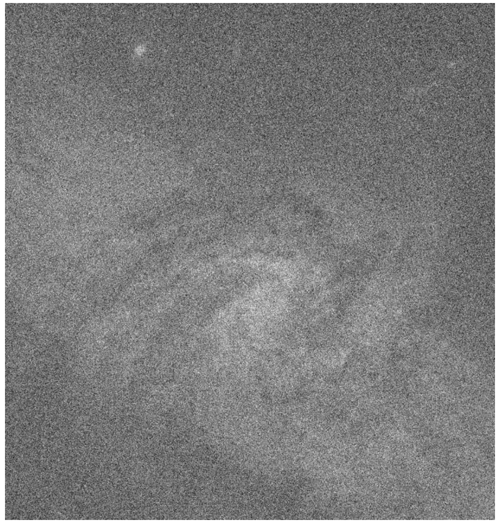


假设噪声服从零均值的高斯分布 $\eta(x,y) \sim N(0, \sigma^2_{\eta(x,y)})$

输入同一物体的*K*副图像 $g_i(x,y),\quad i = 1, 2, \cdots, K$

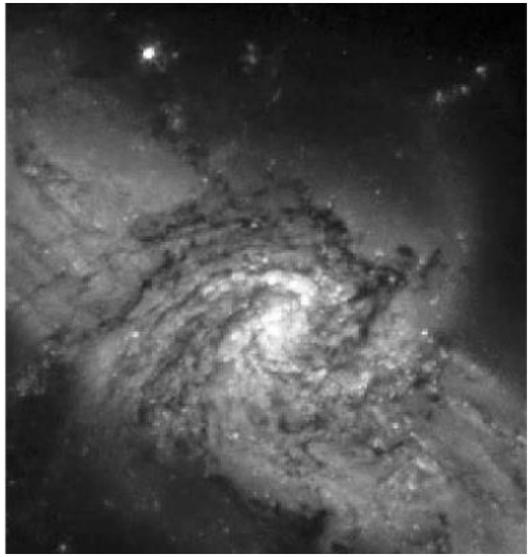
输出图像为 $\overline{g}(x,y) = \dfrac{1}{K}\sum_{i=1}^{K} g_i(x,y)$

期望 $E\left\{\overline{g}(x,y)\right\} = f(x,y)$　　**方差** $\sigma^2_{\overline{g}(x,y)} = \dfrac{1}{K}\sigma^2_{\eta(x,y)}$

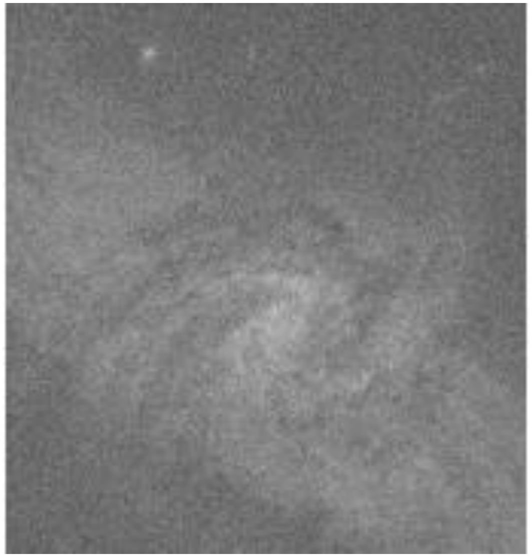


图像降噪

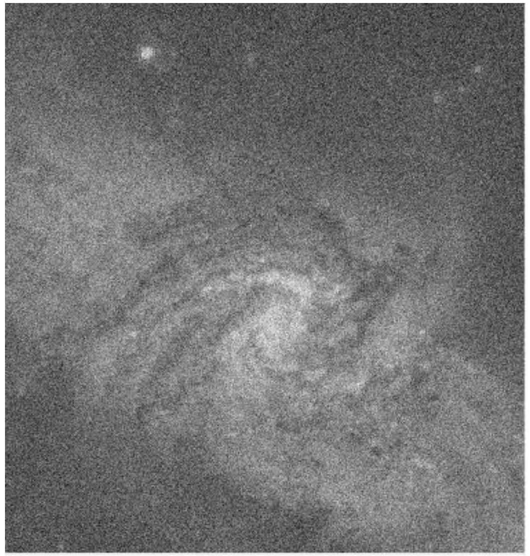
理想图像 （8 bit）



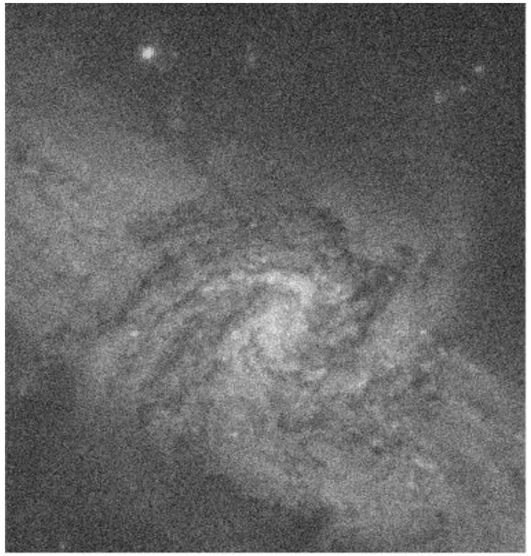
噪声图像 (8 bit)



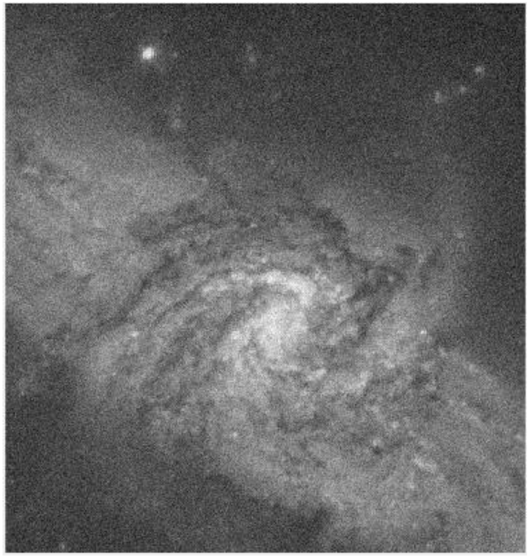
NEX=5



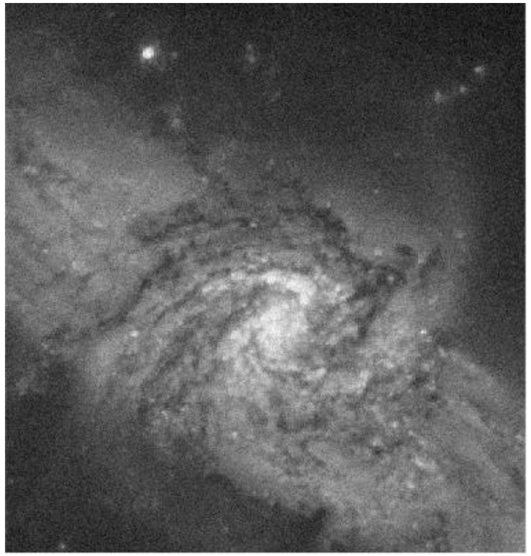
NEX=10



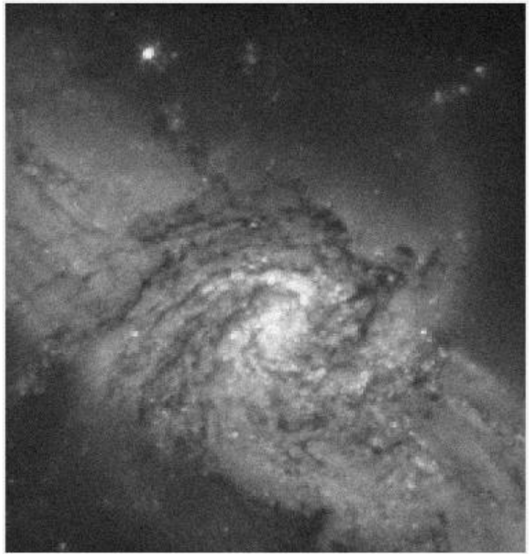
NEX=20

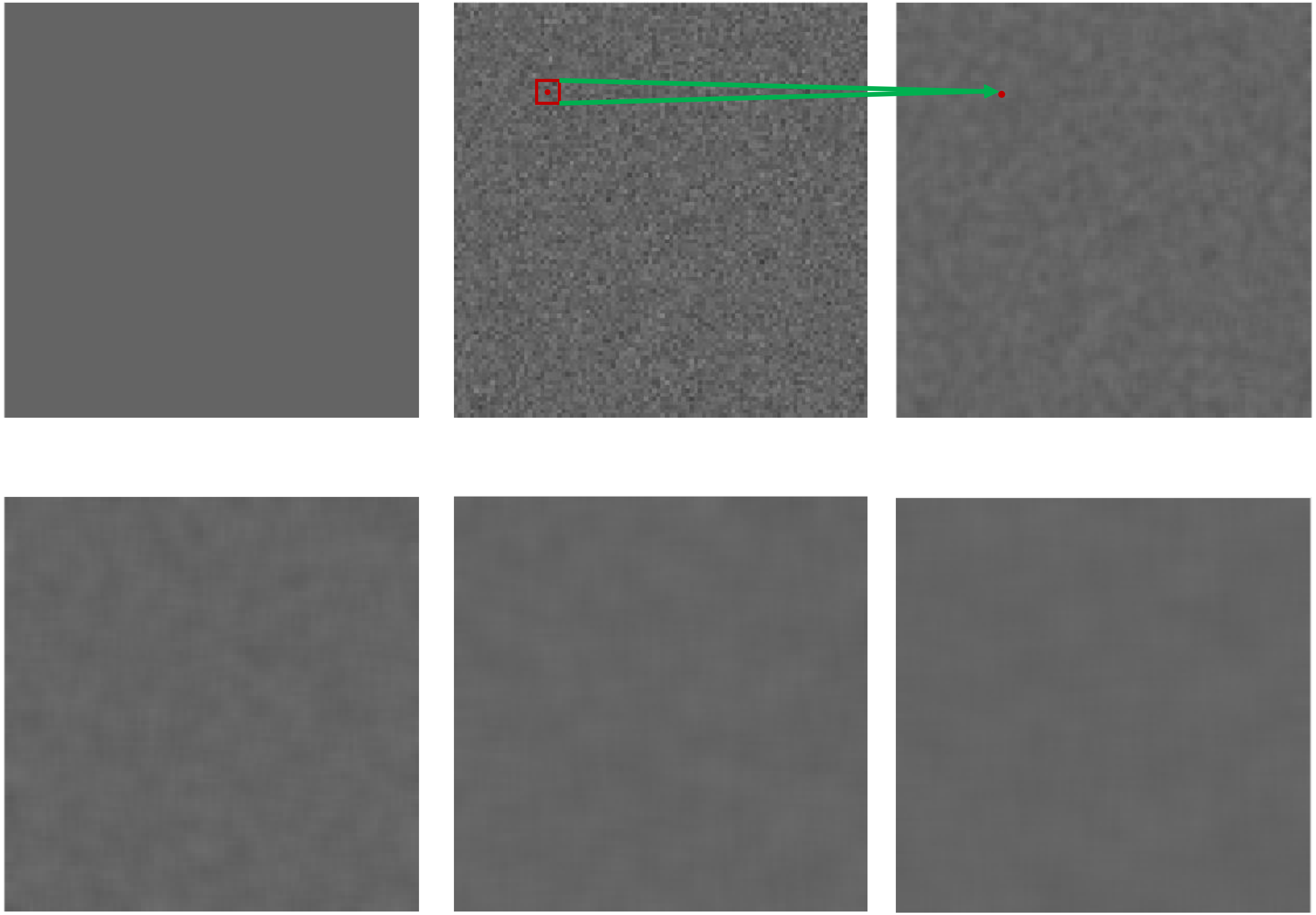


NEX=50

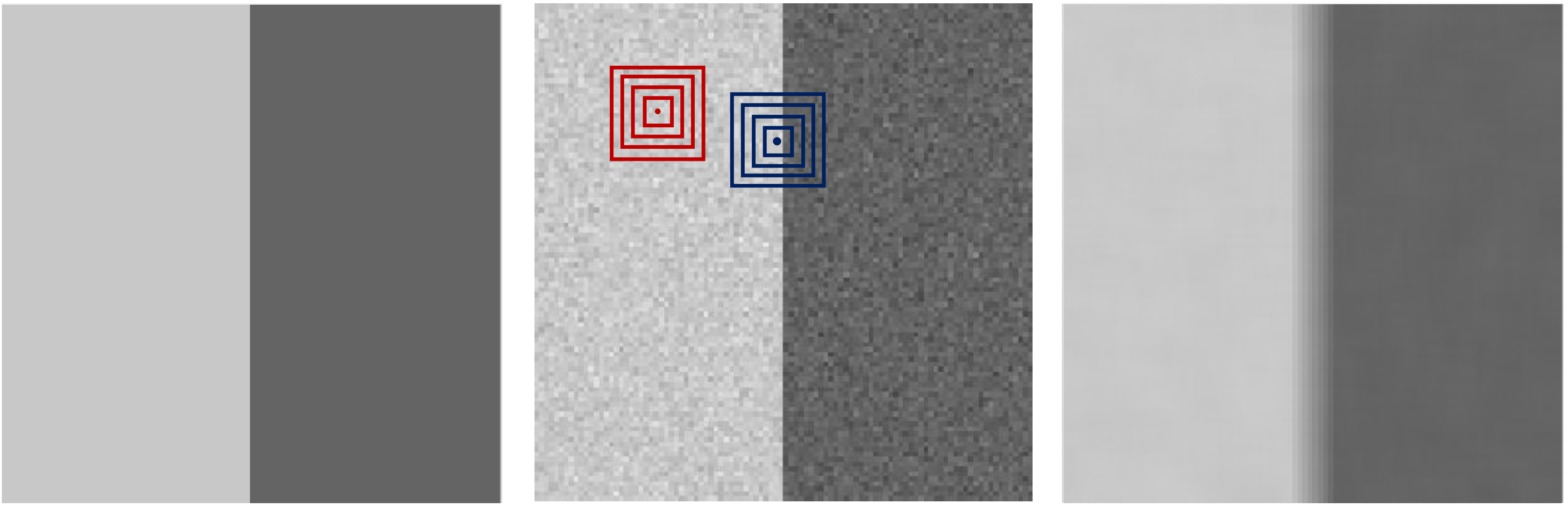


NEX=100





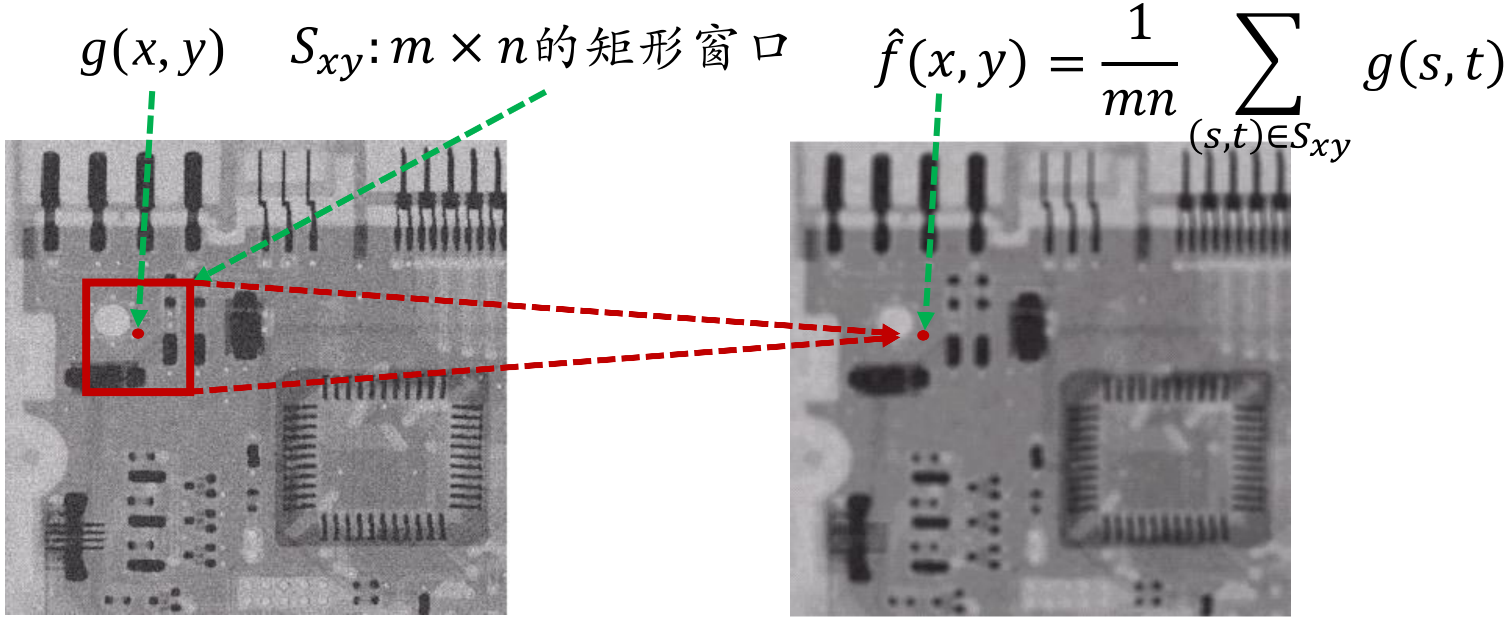
$I = 100, \ \sigma = 10$



$\sigma = 10$

5.3 只存在噪声的复原 —— 空间滤波

- **均值滤波器**
  - **算术均值** $\hat{f}(x,y) = \dfrac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$



$g(x,y)$

S_{xy}: $m \times n$的矩形窗口

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

The filter size is 7 × 7

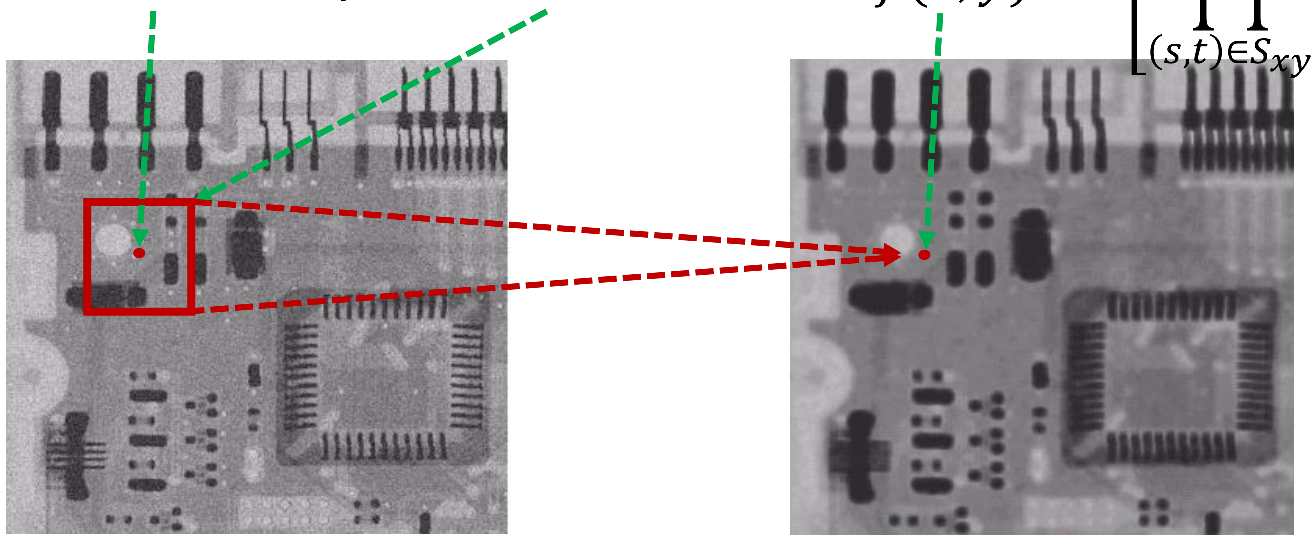
图像细节模糊

图像噪声降低

5.3 只存在噪声的复原 —— 空间滤波

- **均值滤波器**
  - **几何均值**

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{MN}}$$



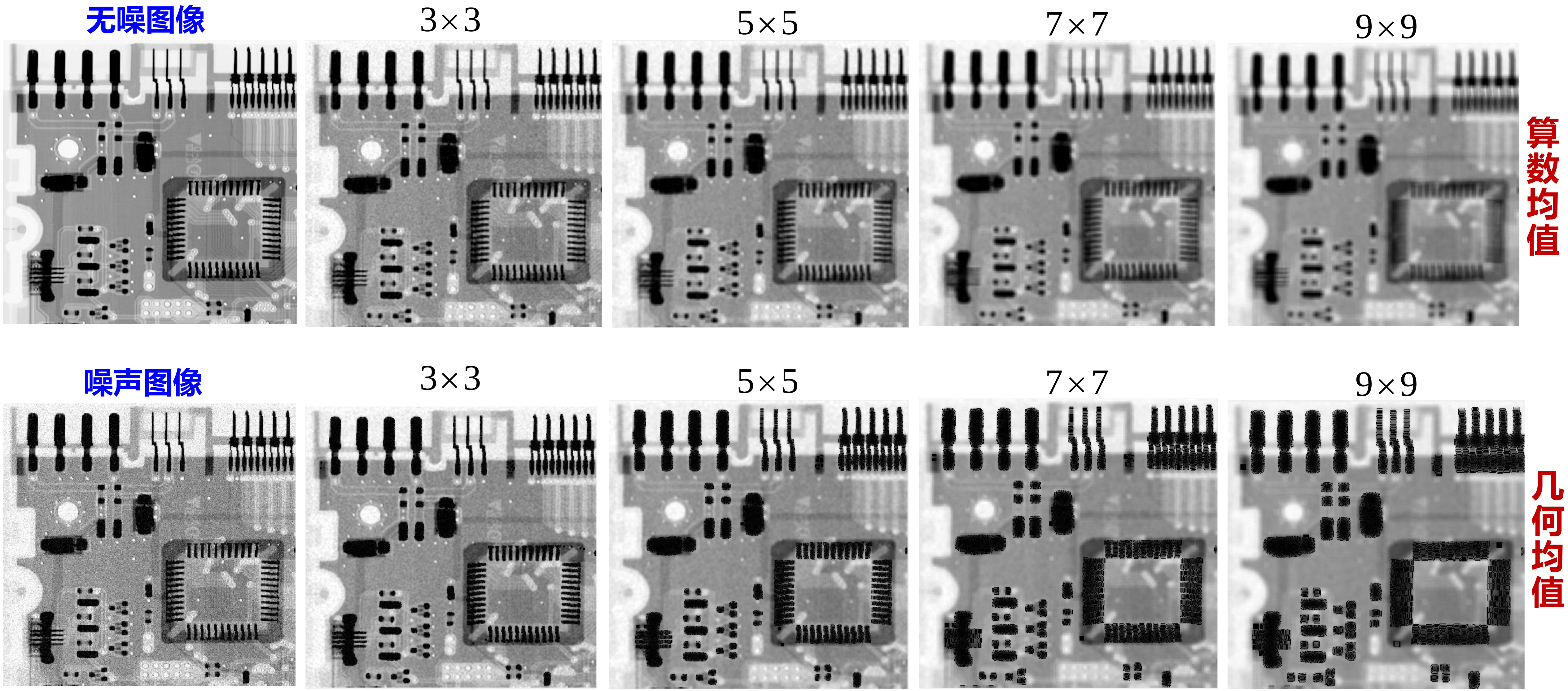
$g(x,y)$

$S_{xy}: m \times n$的矩形窗口

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{MN}}$$

The filter size is 7 × 7

• 算数均值vs几何均值



5.3 只存在噪声的复原 —— 空间滤波

- **自适应均值滤波**

$$\hat{f}(x,y) = w m_L + (1-w) g(x,y)$$

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

算术均值

$$w = \frac{\sigma_\eta^2}{\sigma_L^2}$$

σ_η^2 **整幅图像像素值的方差**

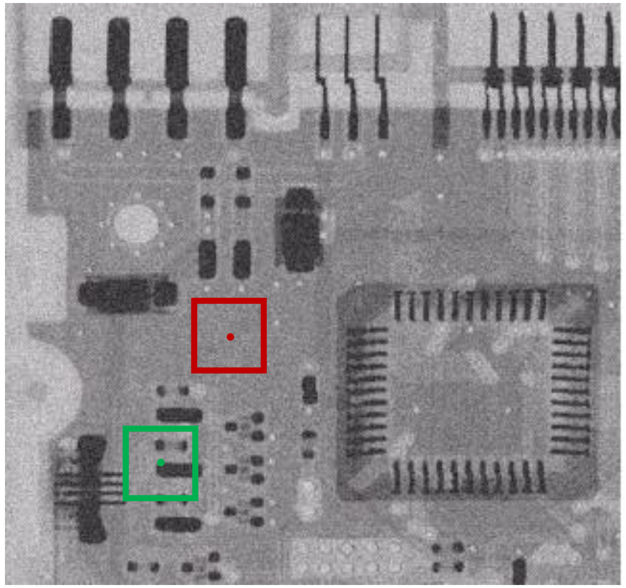
σ_L^2 S_{xy}**内所有像素值的方差**

分析

$\sigma_\eta^2 = \sigma_L^2$ **均匀区域** $\hat{f}(x,y)=m_L$

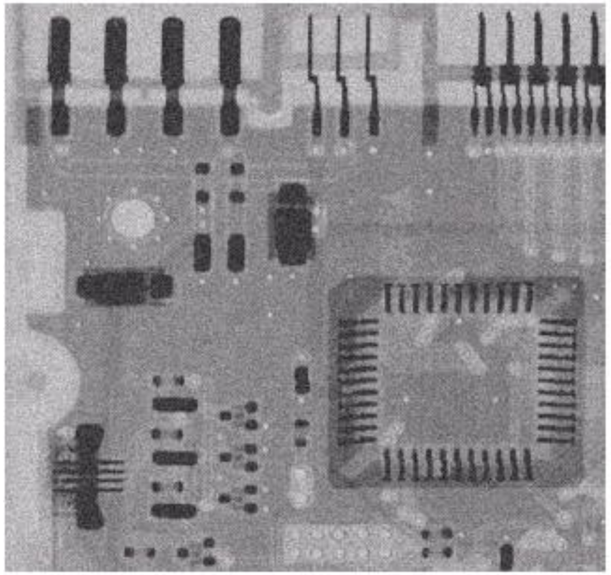
$\sigma_\eta^2 < \sigma_L^2$ **细节或是边缘**

$\sigma_\eta^2 > \sigma_L^2 \rightarrow \frac{\sigma_\eta^2}{\sigma_L^2}=1$ $\hat{f}(x,y)=m_L$

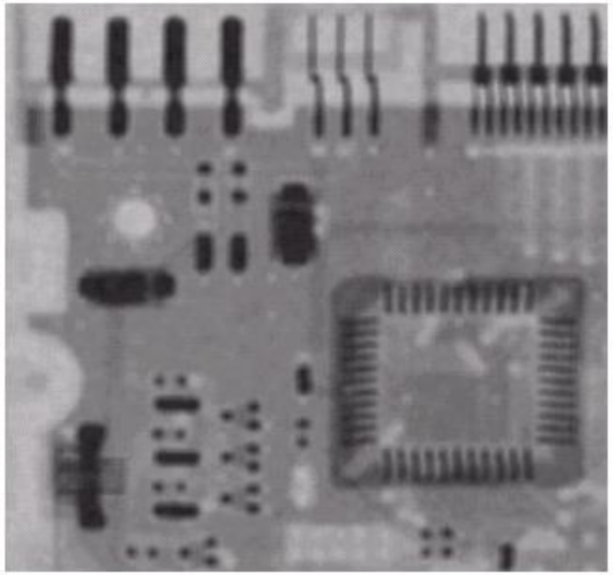


5.3 只存在噪声的复原 —— 空间滤波

高斯噪声图像

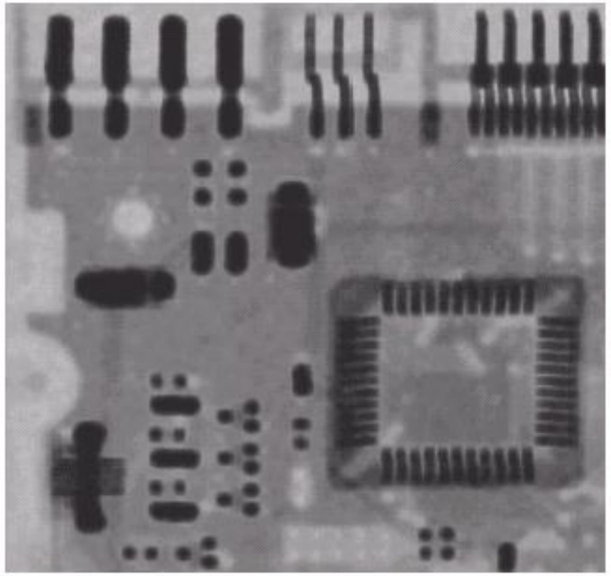


算术均值



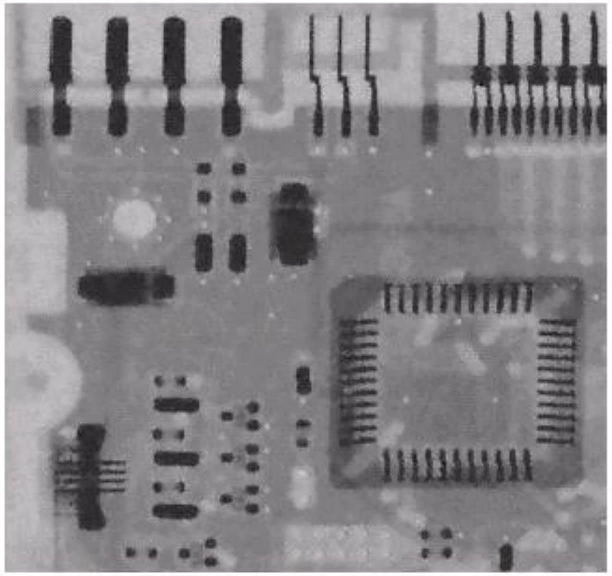
7×7

几何均值



7×7

自适应均值



7×7

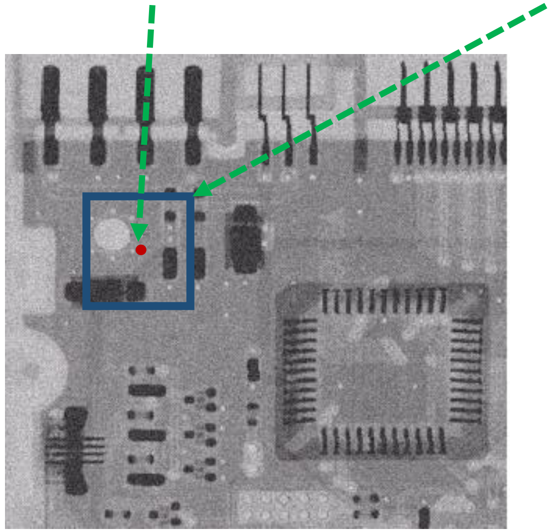
5.3 只存在噪声的复原 —— 空间滤波

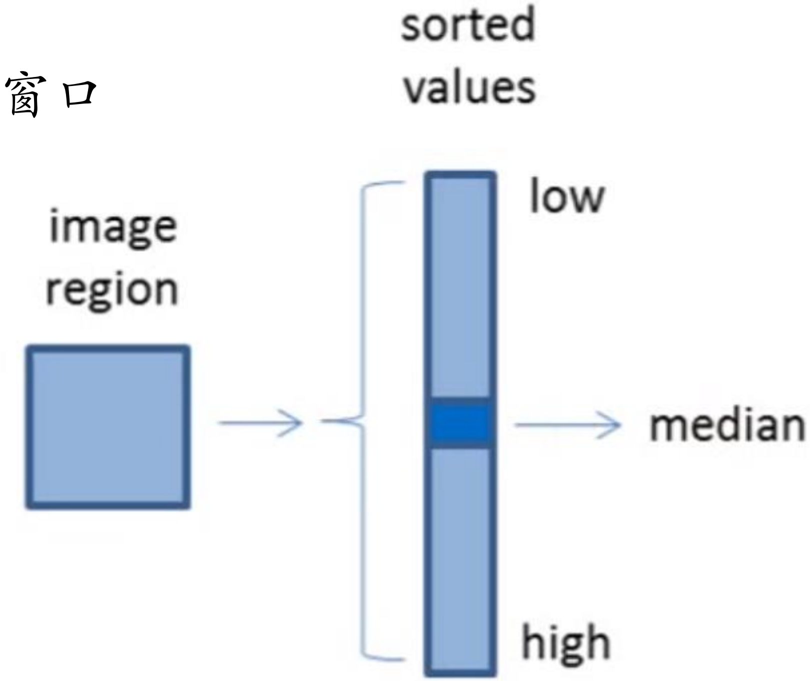
- **统计排序滤波器**
  - **中值滤波** $\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}}\left\{g(s,t)\right\}$

对脉冲噪声尤其有效！

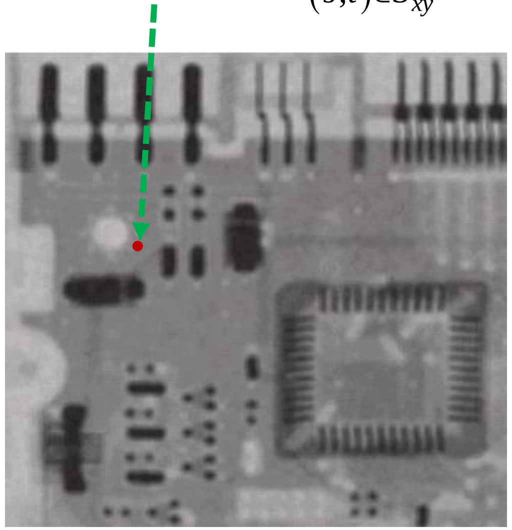
比相同尺寸的均值滤波器模糊要轻

$g(x,y)$ $S_{xy}: m \times n$的矩形窗口





$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}}\left\{g(s,t)\right\}$

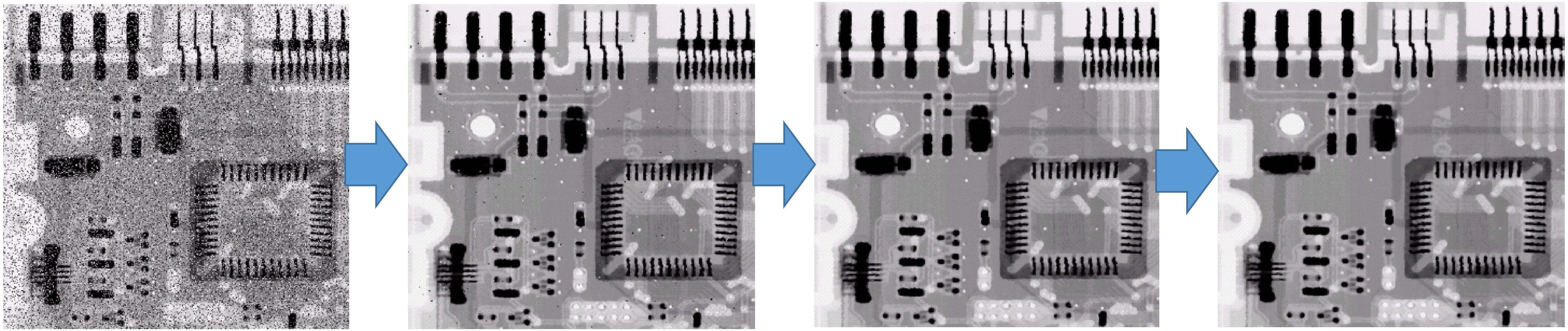


The filter size is 7 × 7

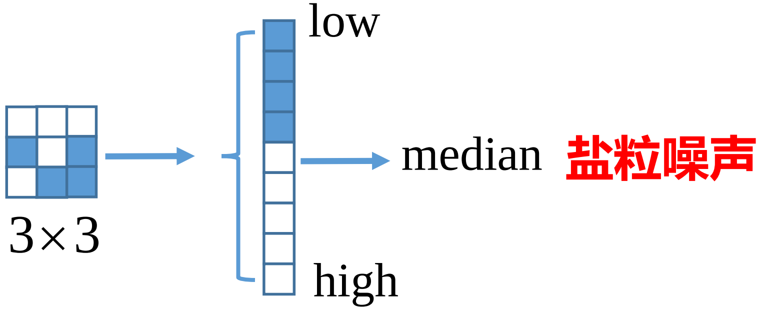
5.3 只存在噪声的复原 —— 空间滤波

- **中值滤波**

椒盐噪声图像　　3×3　　3×3　　3×3



$P_a = P_b = 0.1$



5.3 只存在噪声的复原 —— 空间滤波

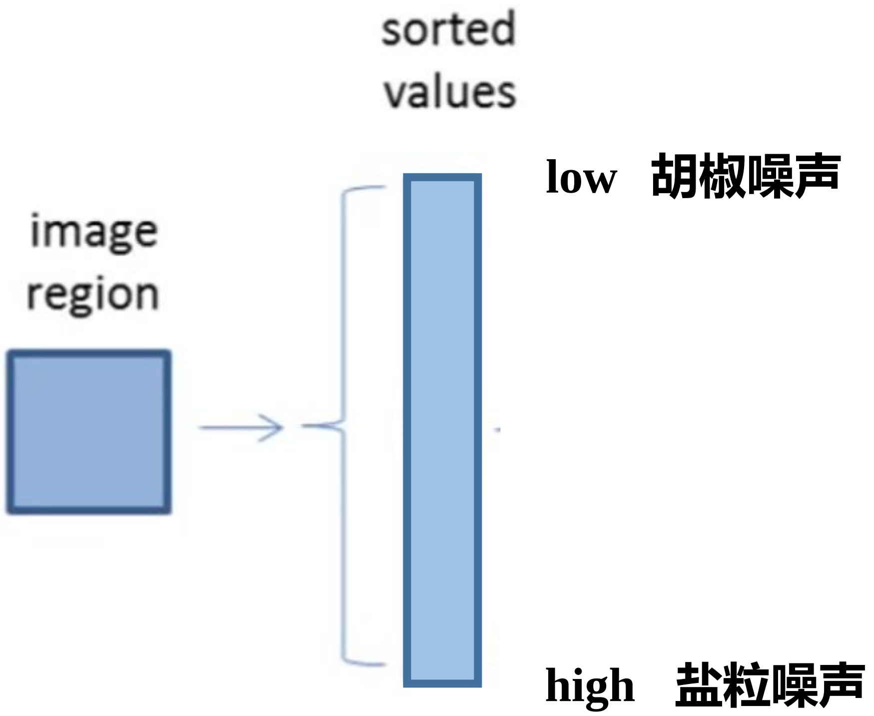
- **统计排序滤波器**
  - **最大值和最小值滤波器**

$$\hat{f}(x,y)=\max_{(s,t)\in S_{xy}}\{g(s,t)\}$$

■ 适用于胡椒噪声

$$\hat{f}(x,y)=\min_{(s,t)\in S_{xy}}\{g(s,t)\}$$

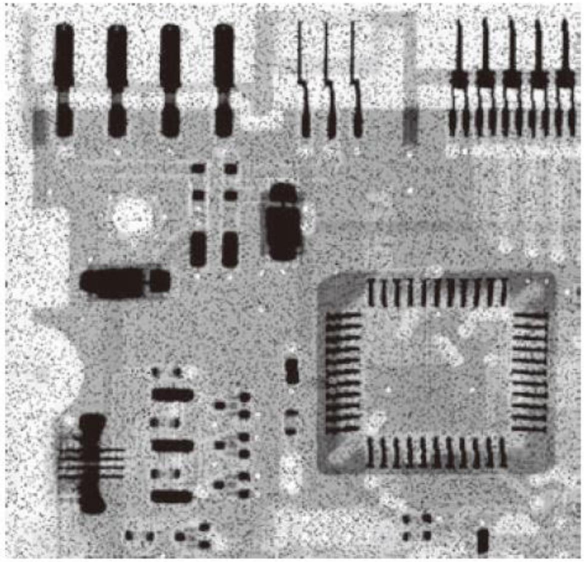
■ 适用于盐粒噪声



5.3 只存在噪声的复原 —— 空间滤波

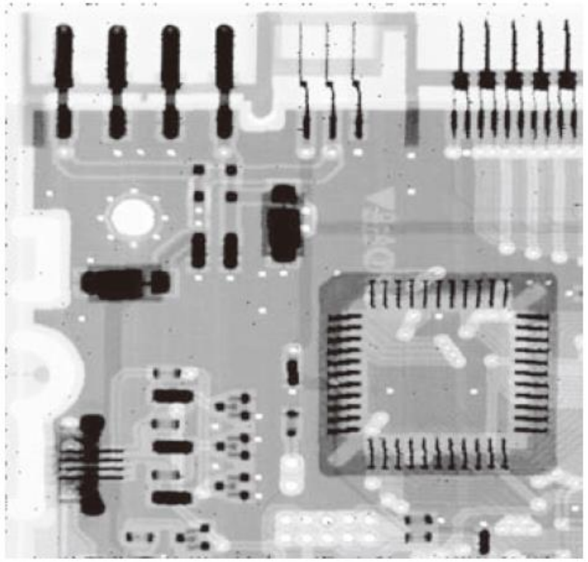
- **最大值和最小值滤波**

胡椒噪声图像

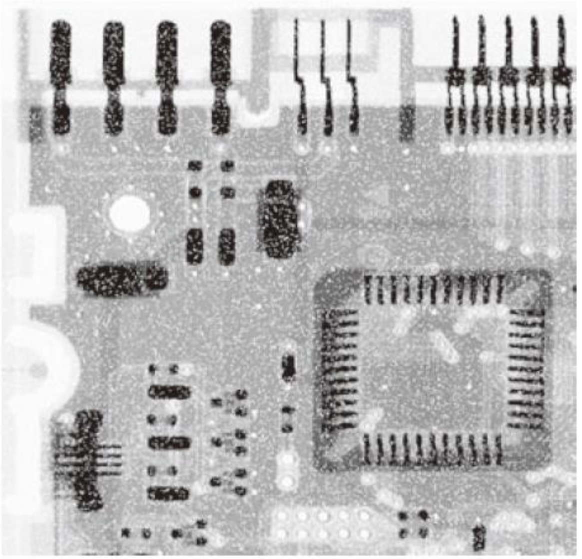


$P_a = 0.1$

最大滤波器

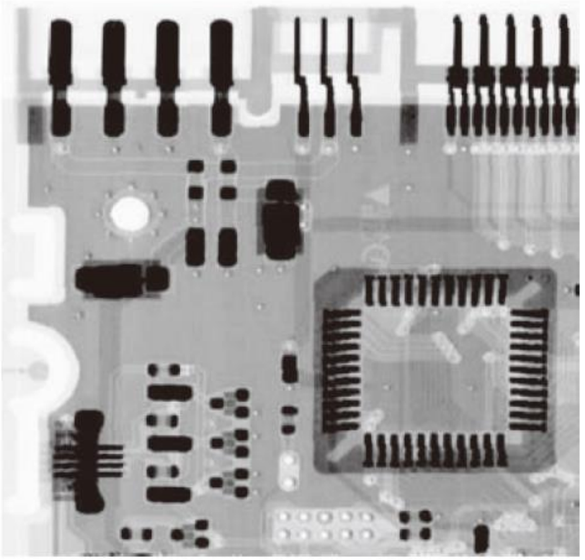


盐粒噪声图像



$P_b = 0.1$

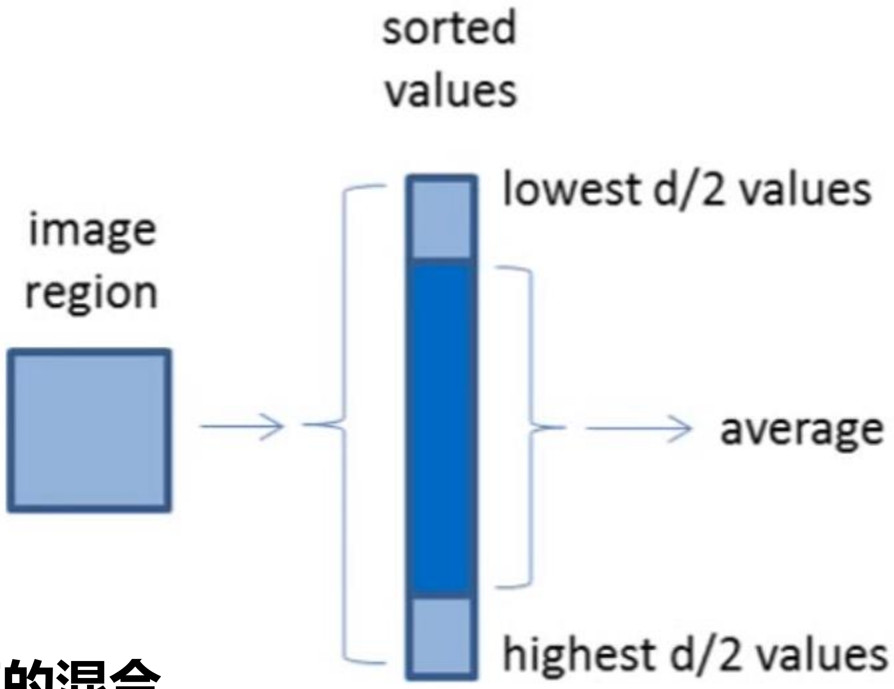
最小滤波器



5.3 只存在噪声的复原 —— 空间滤波

- **统计排序滤波器**
  - **Alpha-trimmed 均值滤波**

$$\hat{f}(x,y)=\frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$



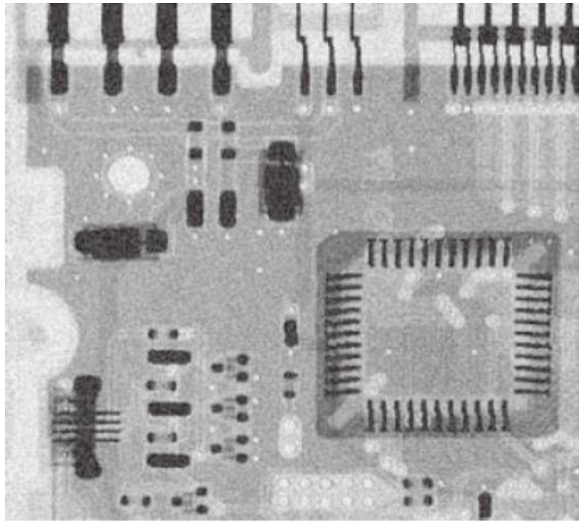
其中 $d\in[0,mn-1]$ **高斯噪声和椒盐噪声的混合**

当 $d=0$ **时** → **算数均值滤波器** **高斯噪声**

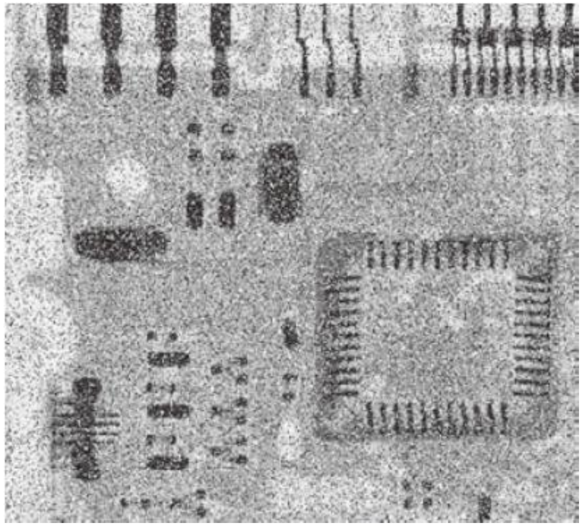
当 $d=mn-1$ **时** → **中值滤波器** **椒盐噪声**

5×5 filter

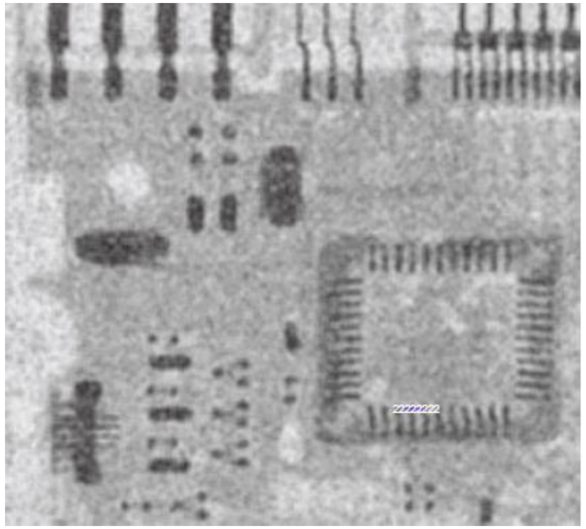
引入均匀噪声的图像



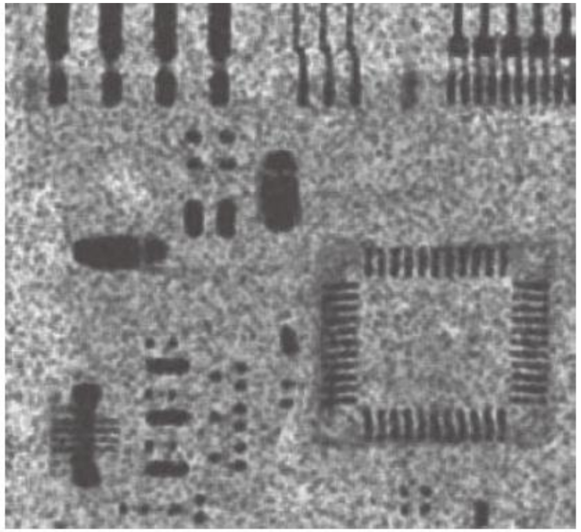
均匀噪声+椒盐噪声



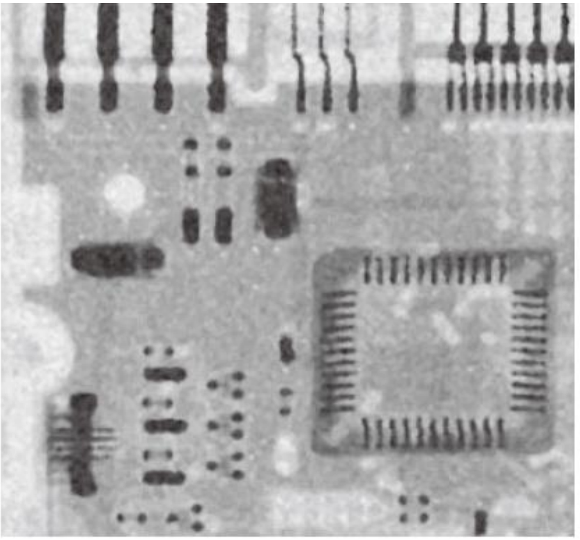
算数均值



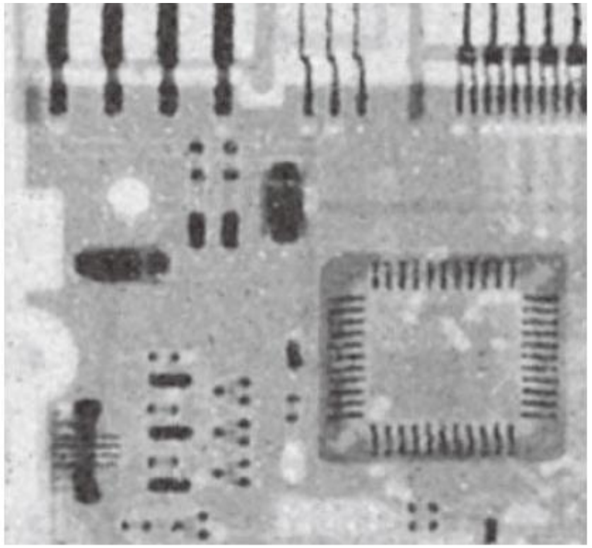
几何均值



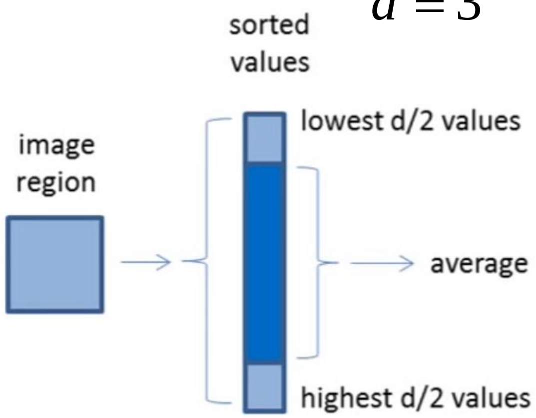
中值滤波



Alpha-trimmed 均值滤波

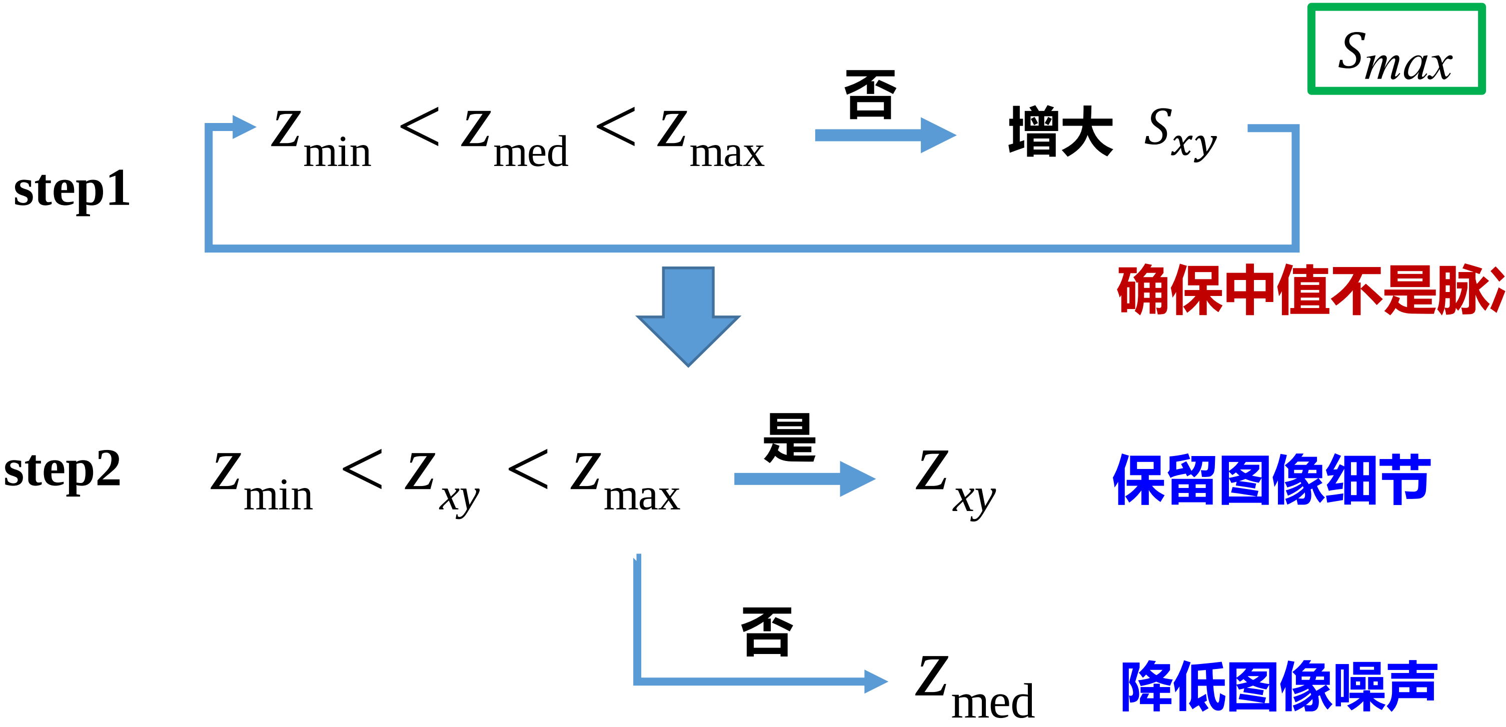


$d = 3$



5.3 只存在噪声的复原 —— 空间滤波

- 自适应中值滤波 Adaptive median Filters



$\boxed{S_{max}}$

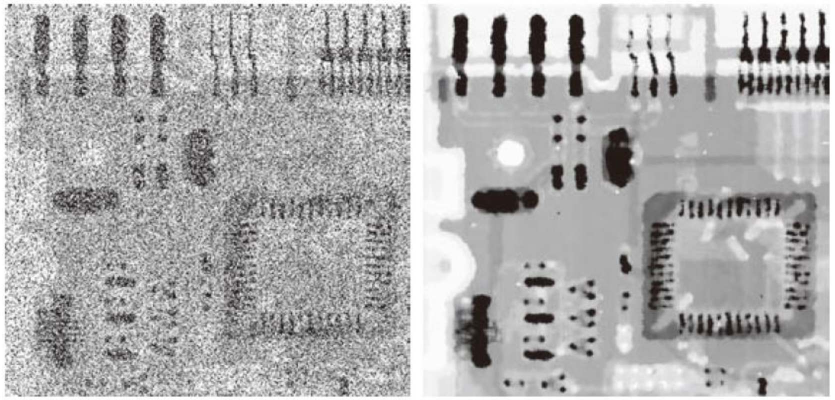
step1 $Z_{min} < Z_{med} < Z_{max} \xrightarrow{否}$ **增大** S_{xy}

确保中值不是脉冲噪声

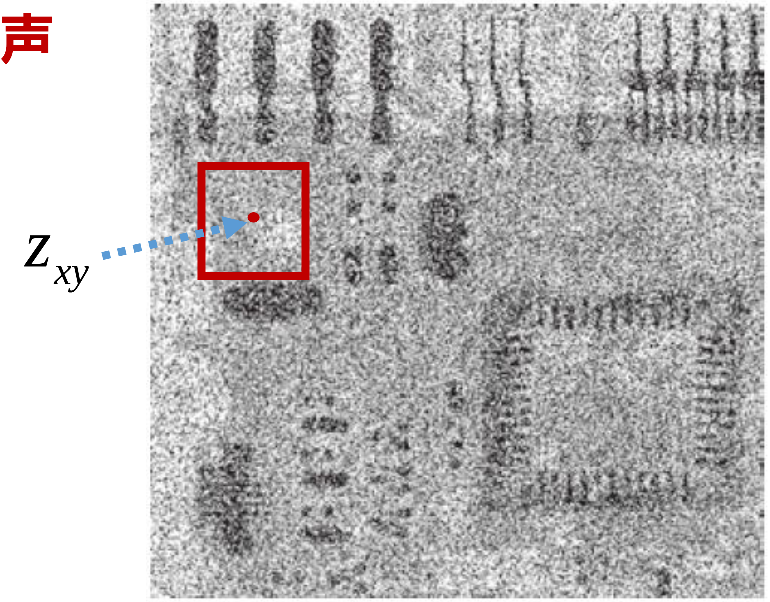
step2 $Z_{min} < Z_{xy} < Z_{max} \xrightarrow{是} Z_{xy}$ 保留图像细节

$\xrightarrow{否} Z_{med}$ 降低图像噪声

$P_s = P_p = 0.25$ median Filters

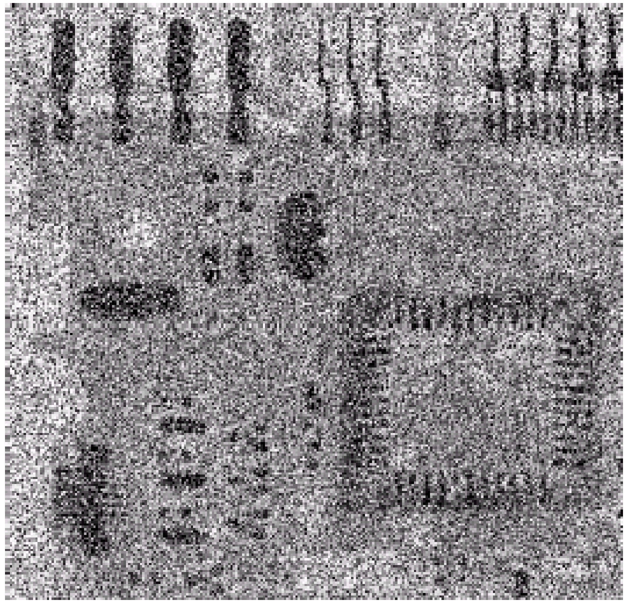


可以处理更大概率的脉冲噪声！

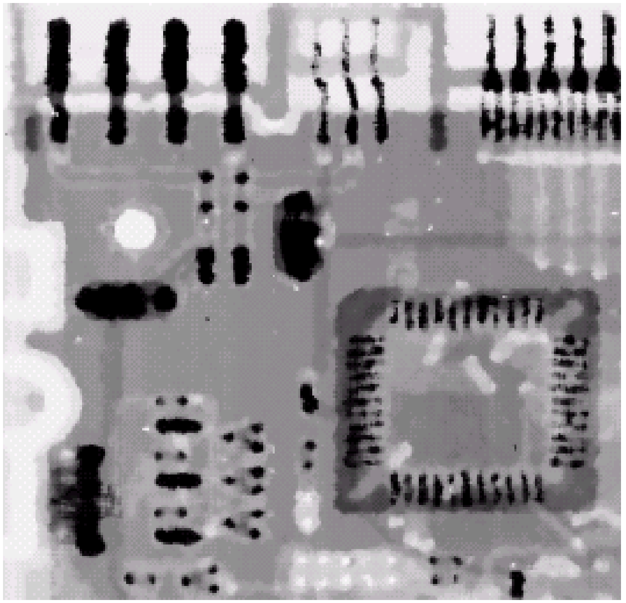


5.3 只存在噪声的复原 —— 空间滤波

椒盐噪声 $P_s = P_p = 0.25$

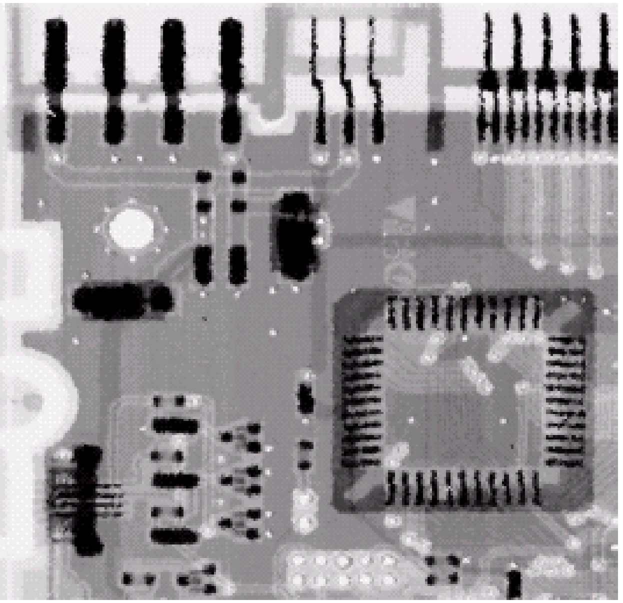


中值滤波



7×7

自适应中值滤波



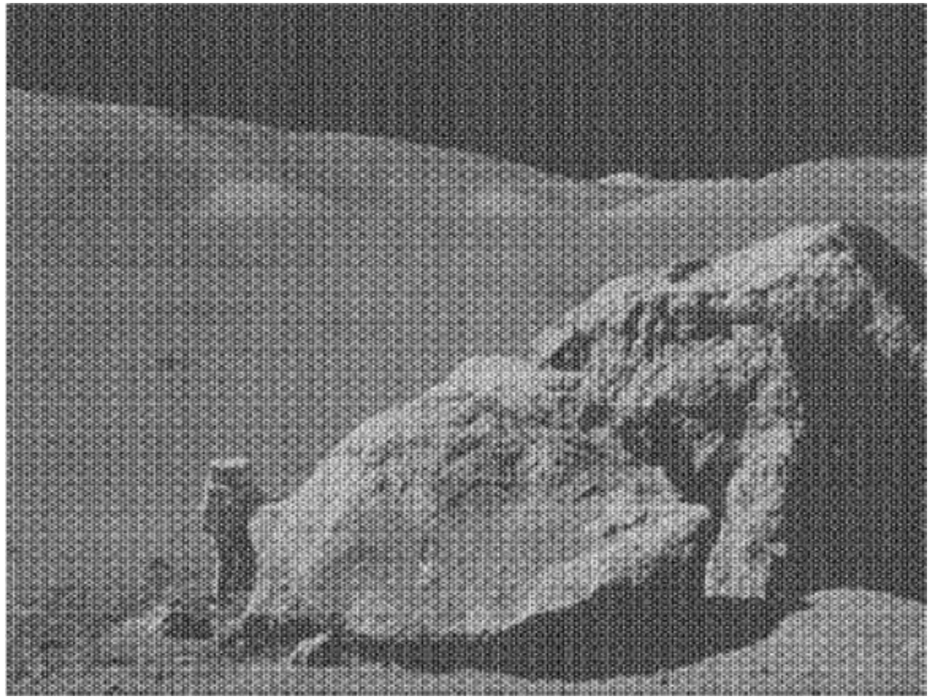
$S_{max}=7\times7$

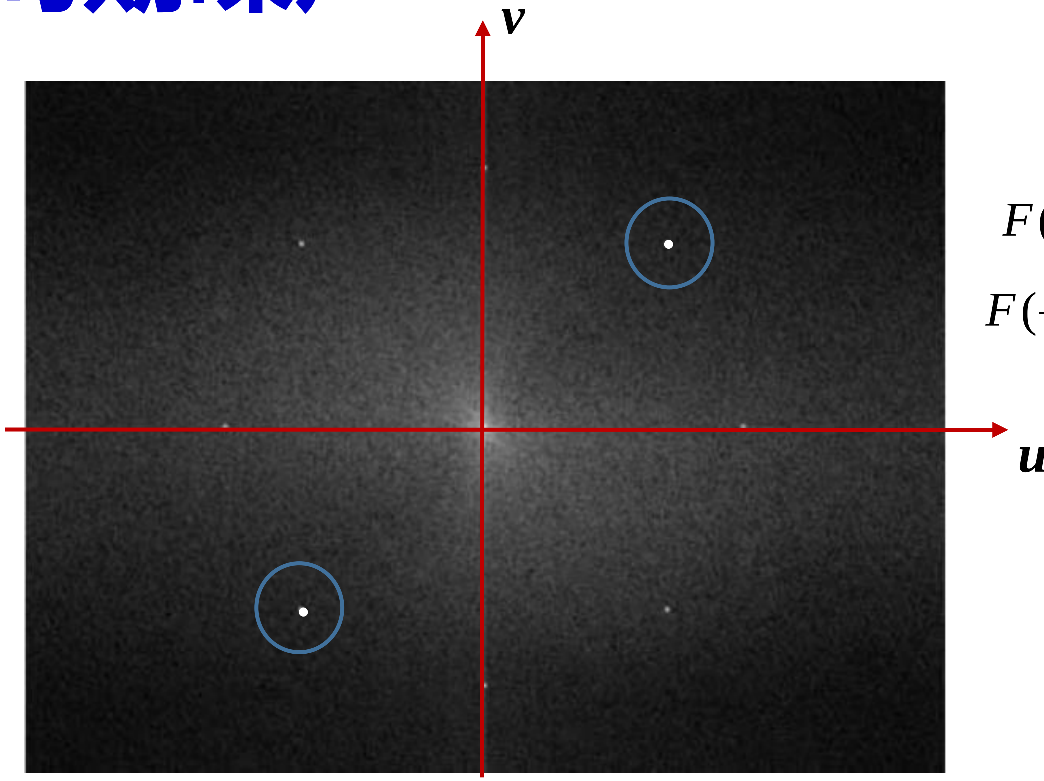
主要内容

- 图像退化/复原过程的模型
- 噪声模型
- 空间域滤波方法
- 频率域滤波方法
- 退化函数的估计
- 逆滤波
- 维纳滤波

5.4 用频率域滤波消除周期噪声

每对共轭脉冲对应于一个正弦波





$$F(u_0, v_0) = a + bi$$

$$F(-u_0, -v_0) = a - bi$$

$$f(x,y) = 2a\cos\left(\frac{u_0 x}{M} + \frac{v_0 y}{N}\right) - 2b\sin\left(\frac{u_0 x}{M} + \frac{v_0 y}{N}\right)$$

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v) e^{j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

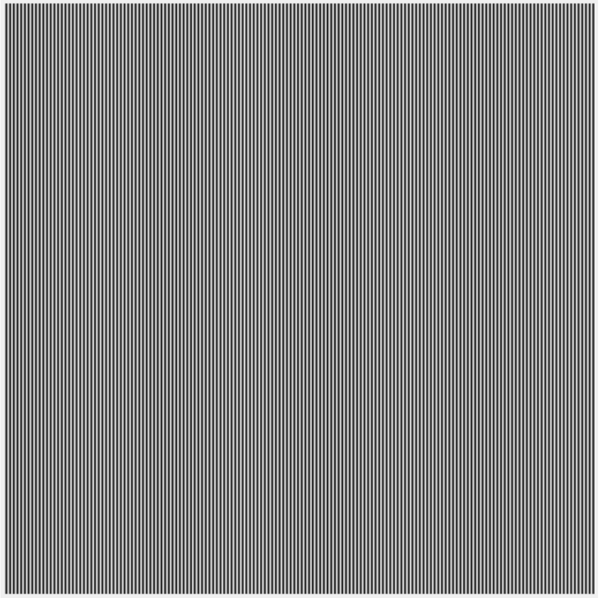
$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

5.4 用频率域滤波消除周期噪声

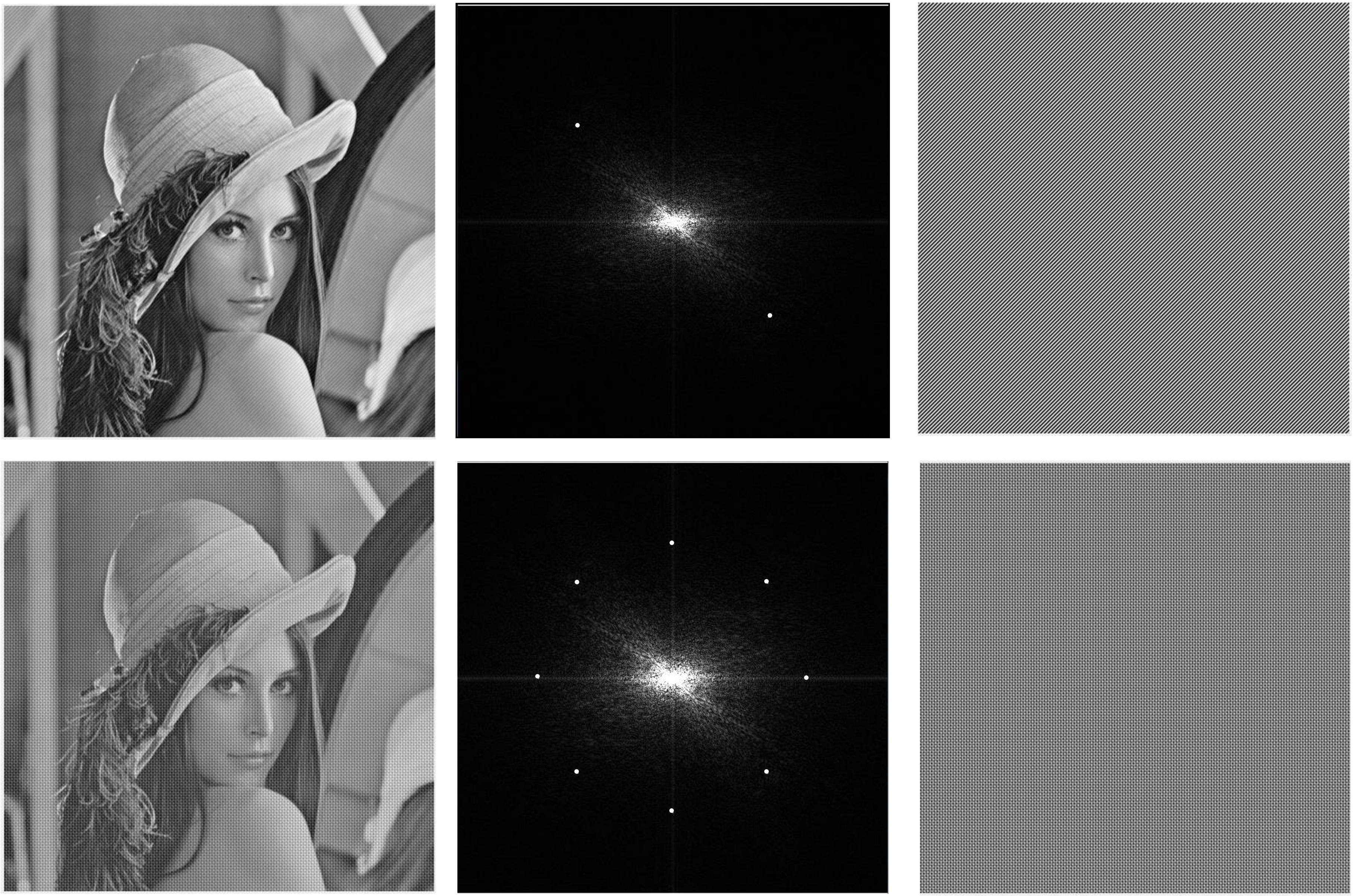
$$F(u_0,v_0)=a+bi$$

$$F(-u_0,-v_0)=a-bi$$

$$f(x,y)=2a\cos\left(\frac{u_0x}{M}+\frac{v_0y}{N}\right)-2b\sin\left(\frac{u_0x}{M}+\frac{v_0y}{N}\right)$$



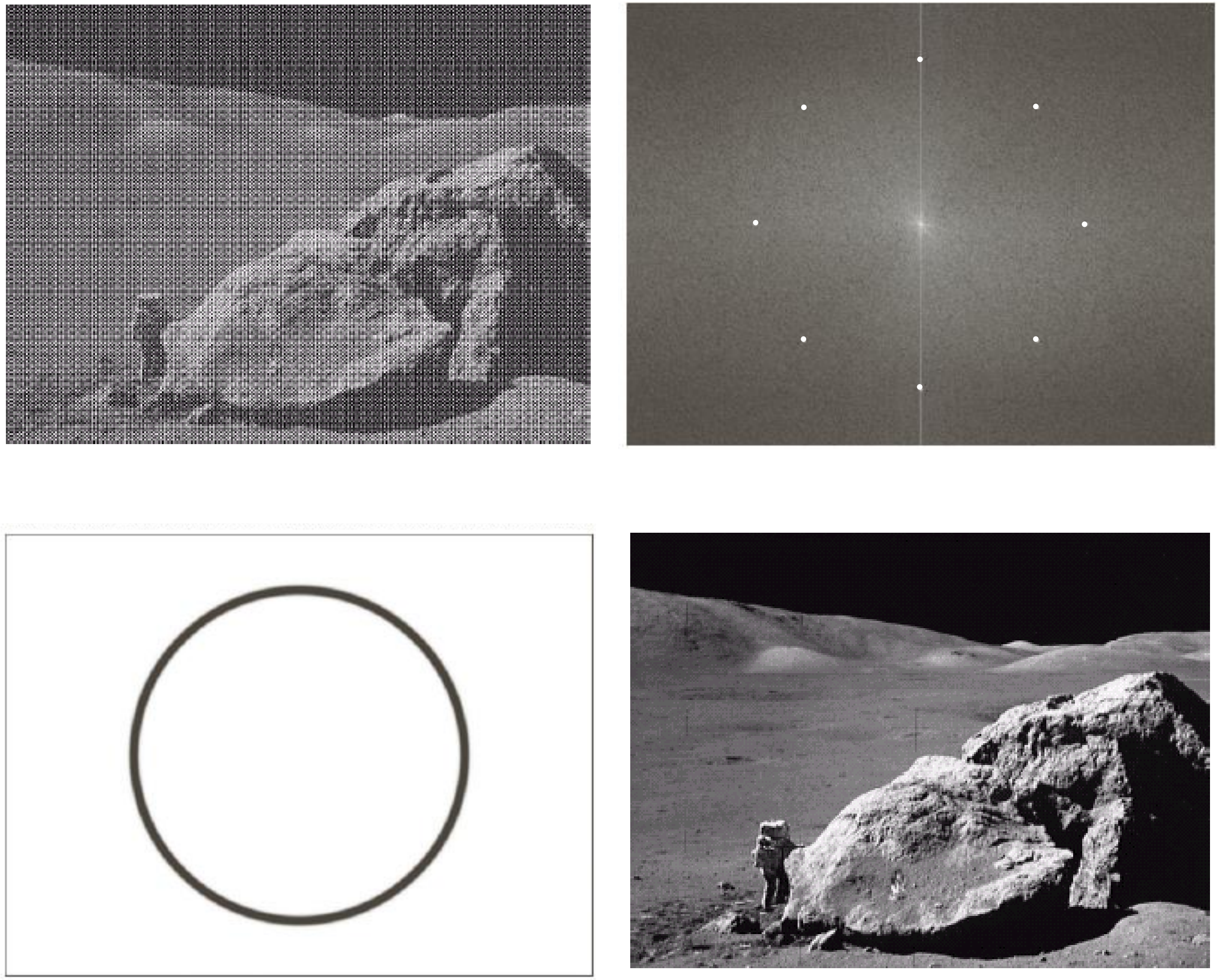
5.4 用频率域滤波消除周期噪声

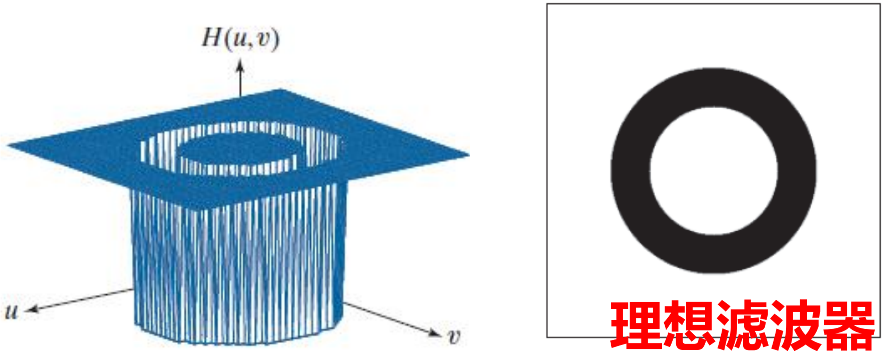


用频率域技术可以有效分析并滤除周期噪声

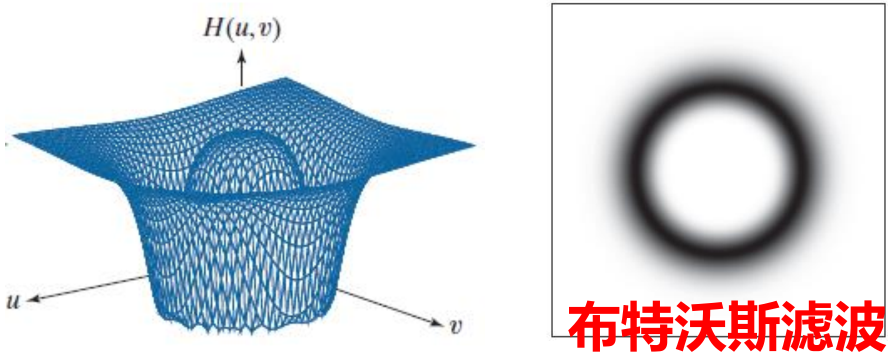
选择性滤波器

5.4 用频率域滤波消除周期噪声——带阻滤波器

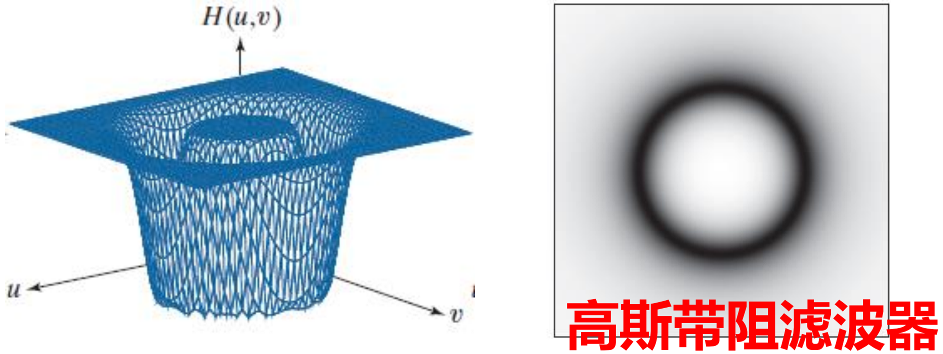




理想滤波器



布特沃斯滤波器



高斯带阻滤波器

5.4 用频率域滤波消除周期噪声——限波滤波器